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SPECIAL TYPE OF TOPOLOGICAL SPACES USING [0, N)

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PREFACE

In this book authors for the first time introduce the notion of special type of topological spaces using the interval [0, n). They are very different from the usual topological spaces. Algebraic structure using the interval [0, n) have been systemically dealt by the authors. Now using those algebraic structures in this book authors introduce the notion of special type of topological spaces. Using the super subset interval semigroup special type of super interval topological spaces are built.

Several interesting results in this direction are obtained. Next six types of topological spaces using subset interval pseudo ring semiring of type I is built and they are illustrated by examples. Strong Super Special Subset interval subset topological spaces (SSSS-interval subset topological spaces are constructed using the algebraic structures semigroups, pseudo groups or semirings or pseudo rings. These topological spaces behave in a very different way. Several problems are suggested at the end of each chapter of the book.

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Chapter One

INTRODUCTION

Study of algebraic structures using [0, n) has been carried out extensively in [26]. Here we study algebraic structures on subsets of [0, n). When we speak of subsets of [0, n) and also of subsets which include also intervals. Two types of subsets can be built. We give algebraic structures on them.

In chapter II we introduce all types of algebraic structures on subsets on [0, n) and on subsets which include also the intervals.

We also study and develop subsets topological spaces of different types.

On every collection of subsets in [0, n) excluding intervals we have built 5 types of semigroups with min operation, max operation, $\cap$, $\cup$ and $\times$. We have on the subset collection of [0, n); the addition operation yields a group.

We also have three types of semirings using the two binary operations {min, max} or $\{\cup, \cap\}$ or $\{\text{min}, \times\}$. In case of the pair of binary operations $\{\text{min}, \times\}$ we see the subset interval semiring is a pseudo semiring.

We see the same can be defined in case of subsets of [0, n) which include the intervals. {S([0, n), ×, min} is a pseudo subset interval semiring. All these algebraic structures are of infinite cardinality.

Now these subsets with the operation + and × is a pseudo ring. We call this as a pseudo ring as the distributive law $a \times (b + c) \neq a \times b + a \times c$ in general for a, b, c ∈ {that pseudo subset interval ring}.

We using these algebraic structures build subset semigroup topological spaces of 3 types, subset interval group topological spaces of three types, subset interval semiring topological spaces of six types, pseudo subset interval ring semiring topological spaces of six types. We obtain several results related with them.

Finally these topological spaces behave in a very different way from the usual topological spaces.

Chapter Two

ALGEBRAIC STRUCTURES ON SUBSETS OF [0,n)

In this chapter we introduce algebraic structures on the subsets of [0, n). In this first place subsets of [0, n) are of either intervals of the form (a, b) or [a, b] $b \neq n$ or x, $x \in [0, n)$ where $x \neq [x, x)$, or [x, x] or (a, b] $b < n$ or [a, b). We know we cannot say $a \leq b$ for this $\leq$ is not compatible with addition and product mod n.

We denote by S([0, n)) = {All subsets of [0, n) = {x, $x \in [0, n)$, [a, b] ($b \neq a$), (a, b] ($b \neq a$), (a, b), a, b $\in$ [0, n)}, clearly $(a, a) \neq a$. We now define algebraic operations on S ([0, n)), get a complete algebraic structure like a semigroup, group or lattice or a pseudo ring and so on.

We first give min operation. We can find given any x, y, $x \neq y$ x,y $\in$ [0, n) smallest or min of x and y.

We see {S([0, n)); min} is a semigroup or a semilattice or an idempotent semigroup.

DEFINITION 2.1: *Let S([0, n)) = {Collection of all subsets of [0, n)}. Define min operation on S([0, n)), {S([0, n)), min} is a* *semigroup defined as the interval subset semigroup which is an idempotent semigroup or a semilattice.*

We illustrate this situation by some examples.

Example 2.1: Let S ([0, 5), min) be the semigroup under min operation.

For [2, 3] $\in$ S([0, 5)) we see min{[2, 3], [2, 3]}= [2, 3].

Let x = (3, 4) and y = (4.1, 4.8)$\in$ S([0, 5)).

min{x, y} = min{(3,4), (4.1,4.8)}

= (min{3,4.1}, min{4, 4.8}) = (3,4) = x.

Let x = [2.1, 4.1) and y = (1.5, 3.6) $\in$ S([0, 5));

min{x, y} = min{[2.1,4.1), (1.5,3.6)}

= (min {2.1, 1.5}, min {4.1, 3.6)) = (1.5, 3.6)
= y $\in$ S([0, 5)).

Let x = [1.2, 3.7] and y = (1.5, 3.6) $\in$ S([0, 5));

min {x, y} = min{[1.2, 3.7], (1.5, 3.6)}
= (min{1.2, 1.5}, min{3.7,3.6}) = [1.2, 3.6).

Let x = 3.1 and y = (3.2, 3.9) $\in$ S([0,5)).

min{x, y}= min{[3.1, 3.1], (3.2, 3.9)} = [min {3.1, 3.2}, min {3.1, 3.9}] = [3.1, 3.1] = 3.1 = x.

Let x = 2.1 and y = (1.5,1.93) $\in$ S([0, 5)).

min{x, y} = min{2.1,(1.5, 1.93)}
= (min{1.5, 2.1}, min{1.93, 2.1}) = (1.5, 1.93) = y.

Let x = [1.4, 1.9) and y = (1.5, 1.93) $\in$ S([0,5)).

min(x,y) = min{[1.4, 1.9), (1.5, 1.93)}
= {min{1.4, 1.5}, min{1.9, 1.93}]
= [1.4 1.9) = x.

min{x,0} = 0 for all x $\in$ S([0,n)).

Thus {S, min} is an idempotent semigroup as min{x,x}= x.

(S([0, n)), min) is an infinite semigroup. Infact {S([0, n)), min)} is a semilattice.

Example 2.2: Let M = {S([0, 12)), min} be the interval subset semigroup o(M) = ∞. M is commutative and is an idempotent semigroup.

Let x = (0.8, 5) and y = (0.1, 4] be in M.

min{x, y}= min{(0.8, 5), (0.1, 4]} = (min{0.8, 0.1),
min{5, 4}) = (0.1, 4] = y $\in$ S.

min{0, x}= 0.

min{0.75, 0.2}= 0.2.

We see min {x, y} $\neq$ x or y.

We see every singleton set is a interval subsemigroup of (S([0, 12), min).

Every pair P = {x, 0}; x $\in$ S([0,12) is also a subsemigroup of S([0, 12)). But every pair is not a subsemigroup.

For let P = {x = [0,9.2) and y = (0.3,5.4)} $\subseteq$ S([0, 12)).

min{x, y} = min{[0, 9.2), (0.3, 5.4])

= [min{0,0.3}, {9.2, 5.4}]
= [0, 5.4] $\in$ S([0, 12)); [0, 5.4] $\neq$ x or y.

Hence the claim.

But min{[0, 9.2], [0.3, 5.4)} = [0, 5.4) $\neq$ x or y.

Thus P_C ={[0, 9.2), (0.3, 5.4), [0, 5.4), [0, 5.4]} $\subseteq$ S([0, 12), min) is a completion of P, P_C is a interval completed subsemigroup of S([0, 12), min).

We see none of these interval subsemigroups are ideals, they are only subsemigroups.

Example 2.3: Let B = (S([0, 19)), min) be the interval semigroup. B is commutative and of infinite order. B has every element 0 $\in$ B to be the least element, however B has no largest element.

We cannot say [0, 18.99] or 18.999 and so on are maximal but none of them is the greatest for the greatest is defined as (19, 19) $\neq$ 19 by convention.

We see M = {[0, 3) [0, 2.9) [0,7)} is a subsemigroup of S([0, 19)).

Now on similar lines we can define max operation on S([0, n)).

DEFINITION 2.2: *Let {S([0, n)), max} be the interval subset semigroup under max.*

We will illustrate this situation by some examples.

Example 2.4: Let {S([0,10)), max} be the interval semigroup.

Let x = (0.5, 2), y = (0.1, 8) $\in$ S([0, 10)), max {x, y} = max{(0.5, 2), (0.1, 8)} = (max{0.5, 0.1}, max{2, 8})= (0.5, 8).

This is the way max operation is performed. However in this case max{x, y} $\neq$ x and not equal to y. Let x = (0.7, 9) and y = (5, 9.5) $\in$ S([0,10)).

max{x,y} = max{(5,9.5),(0.7,9)}
= (max{0.7,5}, max {9,9.5}) = (5,9.5) = y.
max {x,x} = x for all x $\in$ S([0,10)).

Let x = (0.71, 8) and y = (0.5, 9.1) $\in$S([0,10));
max{x, y} = max{(0.71, 8), (0.5, 9.1)}, max {0.71, 0.5}, max{8, 9.1})
= (0.71, 9.1)$\in$ S([0,10)).

Thus the set P= {x, y}is not a subsemigroup only a subset of S([0,10)).

But P_C = {x, y, (0.71, 9.1)} $\in$ S([0, 10)) is a subsemigroup of S([0, 10)).

We see in S(([0,10)), max) every singleton is a subsemigroup. Every finite subset of {S([0,10)), max} can be completed to a subsemigroup.

Infact {S([0, 10), max} is an idempotent semigroup of infinite order which is commulative.

Example 2.5: Let {S([0, 19), max}be the interval semigroup.

Let x = (6, 8.3) and y = (9, 4.2) $\in$S([0, 19))

max{x, y}= max ({(6, 8.3),(9, 4.2})
= (max {6, 9}, max {8.3, 4.2}) = (9, 8.3) $\in$ S([0, 19)).

Now we will consider S([0, n)) to contain the natural class of intervals; so if (5, 7) $\in$ S([0, n)) then (7, 5) $\in$ S([0, n)). That is (a, b) $\in$ S([0, n) implies (b, a) $\in$ S([0, n)) and so on.

Example 2.6: Let {S([0, 24)), max} be the interval semigroup.

Let P = {a_0 = 0, a, ,…, a_n, n can reach infinite, all values in [0, 24)}, that is P does not contain any non trivial interval.

P is a subsemigroup of infinite order, P is not an ideal.

For if $x = 0.5 \in P$ and $b = [0.2, 9.2] \in S([0, 24))$ then
$\max \{0.5, [0.2, 9.2]\}$
$= [\max\{0.5, 0.2\}, \max\{0.5, 9.2\}]$
$= [0.5, 9.2] \notin P$.

Thus P is not an ideal of (S([0, 24)), max).

Finding ideals in {S{[0, n)), min} and {S([0, n)), max} happens to be an open problem.

Let $x = [0.8, 0]$ and $y = [0.7, 9] \in$ (S([0, n)), max).

$\max\{x, y\} = \max\{[0.8, 0],[0.7, 9]\}$
$=[\max\{0.8, 0.7\}, \max\{0, 9\}]$
$=[0.8, 9] \in S\{[0, 24), \max\}$.

Let $x = [23, 2]$ and $y = [5, 14) \in S([0,24))$.
$\max\{x, y\} = \max\{[23, 2], [5, 14)\}$
$= [\max\{23, 5\}, \max\{2, 14\})$
$= [23,14) \in \{S([0, 24)), \max\}$.

Now we can define on S([0, n)) a product ×; (S([0, n)), ×). (S([0, n)), ×) is a commutative semigroup of infinite order.

Example 2.7: Let (S([0, 12), ×) be the interval subset semigroup.

Let $x = [0.3, 7]$ and $y = [2, 0.12] \in S([0, 12))$

$x \times y = [0.3, 7] \times [2, 0.12]$
$= [0.6, 0.84]$.

This is the way × operation is performed; $x \times y = 0$ exist in S.

Let $x = [0, 3]$ and $y = [5, 0] \in S([0, 12))$.
$x \times y = [0, 3] \times [5, 0] = [0, 0] = 0$.

Let x = [0.3, 2] and y = [0.1, 0] $\in$ S([0, 12))

x $\times$ y= [0.3, 2] $\times$ [0.1, 0]
= [0.03, 0].

Thus we can find zero divisons.

Let x = [10, 3] and y =[6, 4] $\in$ S([0, 12));
x $\times$ y=[10, 3] $\times$ [6, 4] = [0, 0] = 0.

Let x = (0.3, 1.2) and y = (0.9, 0) $\in$ S([0, 12));
x $\times$ y = (0.3, 1.2) $\times$ (0.9, 0)
= (0.27, 0) $\in$ S[0, 12).

Let x = [5, 11) and y = (3, 5) $\in$ S([0, 12)).
x $\times$ y= [5, 11) $\times$ (3, 5) = [3, 7] $\in$ S[0, 12).

Let x = [4, 2] and y = [3, 6] $\in$ S([0, 12));
x $\times$ y = [4, 2] $\times$ [3, 6]=[0,0] = 0 $\in$ S[0, 12).

We see for every x $\in$ S([0, 12)); x $\times$ 0 = 0 $\times$ x = 0.

For every x; 1 $\times$ x = x $\times$ 1 = 1 in S([0, 12)) so 1 is the multiplicative identity in S ([0, 12)).

Let x = 5 $\in$ S([0, 12]; we see 5 $\times$ 5 = 1(mod12).

Like wise 6 $\in$ S([0, 12)); 6 $\times$ 6 = 0 (mod 12).

Let x = [6, 4] and y = [2, 3] $\in$ S([0, 12)); x $\times$ y = 0 is a zero divisor in S([0, 12));

Let x= [5, 7] $\in$ S([0, 12)); x $\times$ x = [5, 7] $\times$ [5, 7] = [1, 1] = 1 is a unit in S([0,12)).

Let x = [7, 11] $\in$ S([0, 12)) is a unit in S([0, 12)) and so on.

We can find units, zero divisors and idempotents in S([0, 12)). We have subsemigroups.

If $x = (0.7, 5.1)$ and $y = (8.4, 9) \in S([0, 12))$;
$x \times y = (0.7, 5.1) \times (8.4, 9) = (5.88, 9.9) \in S[0, 12)$.

This is the way operations are performed on S([0, 12)).

Let $x = (2, 5.6]$ and $y = (10, 11) \in S([0, 12))$;
$x \times y = (2, 5.6] \times (10, 11) = (20, 61.6) \pmod{12} = [8, 1.6)$.

Let $x = [0, 2.1)$ and $y = [5, 6) \in S([0, 12))$;
$x \times y = [0, 2.1) \times [5, 6) = [0, 12.6] = [0, 0.6]$.

Thus if in one of the interval a value is covered then we use closed interval or if both of them are closed.

We have subsemigroups and ideals in S([0, 12)).

Example 2.8: Let $P = (S([0, 19)), \times)$ be the interval semigroup. We see P has zero divisors, units and no idempotents $x = (9, 10)$ and $y = (3, 1) \in P$.

$x \times y = (9, 10) \times (3, 1) = (27, 10) = (8, 10) \in P$.

This is the way product is performed.

Let $x = (3.1, 2.9)$ and $y = (7, 10) \in P$.
$x \times y = (3.1, 2.9) \times (7, 10) = (21.7, 29) = (2.7, 10) \in P$.

This is the way product is performed.
$x = [0, 3]$ and $y = [9.111, 0] \in P$;
$x \times y = [0, 3] \times [9.111, 0] = 0$ is a zero divisor in P.

$x = [1, 10]$ and $y = [1, 2] \in P$ is such that
$x \times y = [1, 10] \times [1, 2] = 1$.

Thus P has units.

Study of P for ideals, subsemigroups are a matter of routine and hence left as an exercise to the reader.

M = {(0, 1), (0, 2), …, (0, 18)} $\subseteq$ P is a subsemigroup of P
N = {[0, 1], [0, 2], …, [0, 18]} $\subseteq$ P is a subsemigroup of P.

Both M and N are of finite order. We see both of them are not ideals of P.

Consider W = {[a, b] | a, b $\in Z_{19}$} $\subseteq$ P is a subsemigroup of finite order, x = [9, 10] and y = [2, 2] $\in$ P.
x $\times$ y = [18, 20] = [18, 1] $\in$ W.

W is again a subsemigroup .W is also a finite subsemigroup of P which is not an ideal of P.

Example 2.9: Let M = (S([0, 20)), $\times$); be the interval subset semigroup. W = {[0, 10], (0,0)} is a subsemigroup of order two.

P = {0,[0, 5], [0, 10], [0, 15]} $\subseteq$ M is a subsemigroup.

B ={[0, 2], [0, 4], …, [0, 18], [0, 0]} $\subseteq$ M is a subsemigroup of M. M has several subsemigroups of finite order also. None of these finite order subsemigroups are ideals of M.

Thus we see interval subset semigroups of three types one under min operation, one under max operation and one under $\times$ can be formed.

The next natural question is can we give on S([0, n)) a '+' operation. First we study an example.

Let S = {S([0, 6)), +} be a collection of subsets of [0, 6).

Let x = (3, 4) and y = (4.21, 2) $\in$ S
x + y = (3, 4) + (4.21, 2) =(7.21, 6) = [1.21, 0].

We see if x = (2.113, 1.21) $\in$ S then we have a unique

$y = (3.887, 4.79) \in S$ such that
$x + y = (2.113, 1.21) + (3.887, 4.79) = (6, 6) = (0, 0) = 0$.
Thus (0, 0) is additive identity.

Let $a = (0.37, 4.19) \in S$ and $b = (5.63, 1.81) \in S$ we see $a + b = b + a = (0, 0)$. a is the inverse of b and b is the inverse of a.

Hence S([0, 6), +) is a group under +.

Now we technically make the definition of interval group.

DEFINITION 2.3: *Let {S([0, n)), +} = S be the collection of all subsets of [0, n) under the binary operation +.*

Clearly for every $x, y \in S$ we see $x + y \in S$.
Further $x + y = y + x$ for all x,y S. $(0, 0) = 0$ is the identity of S with respect to +.

Now for every $x \in S$ we have a unique $y \in S$ such that $x + y = y + x = (0, 0) = 0$ called the additive inverse of S.

S is a commutative group of infinite order.

S is defned as the interval subset group or pseudo group of [0, n) under addition modulo n.

We will just illustrate by an example or two this situation.

Example 2.10: Let S = {S([0,10)), +)} be the subset interval group under '+'.

For every x, y in S; $x + y = y + x \in S$. $0 + x = x + 0 = x$ for all $x \in S$.

Further if $x = (0, 7.2)$ and $y = (5, 4) \in S$ we see
$x + y = (0, 7.2) + (5, 4) = (5, 1.2) \in S$. Thus it is clear why the authors demand $(b, a) \in S$ for all $a, b \in S$.

Let x = (4, 6) and y = (6, 7.2) $\in$ S.

$$\begin{aligned} x + y &= (4, 6) + (6, 7.2) \\ &= (4 + 6, 6 + 7.2) \\ &= (0, 3.2) \in S. \end{aligned}$$

Let x = (3, 4.3) then y = (7, 5.7) $\in$ S is such that
x + y = (3, 4.3) + (7, 5.7) = 0.

Example 2.11: Let S ={S[0, 15), +} be the subset interval group.

P ={Z_{15}, +} is a subgroup of finite order.

Let S_1 = {[0, 15), +} be the subset interval subgroup of S.

Let x = (0.3, 7) and y = (10, 8) $\in$ S; x + y = y + x = (10.3, 0) $\in$ S. This is the way + operation is done on S.

Now P_1 = {[0, 0], [0, 1], …, [0, 14], +} $\subseteq$ S is a subgroup of finite order.

S has both finite and infinite interval subgroups.

P_2 = {[0, 0], [1, 0], [2, 0], …, [14, 0], +} is again a interval subset subgroup.

Study in this direction is important and interesting.

Example 2.12: Let S = {S([0, 26)), +} be the subset interval pseudo group. o(S) = ∞. S is commutative. S has both finite and infinite subgroups.

As S is commutative and several classical results related with finite groups cannot be studied.

Example 2.13: Let S ={S([0,43)), +} be the subset interval pseudo group. S has both subgroups of finite and infinite order.

Now we try to build algebraic structures using S([0, n)) with two binary operations.

{S([0, n)), min, max} is an interval semiring of infinite order.

Clearly {S([0, n)), min} and {S([0, n)), max} are commutative semigroups and min and max are distributive over each other.

{S([0, n)), min, max} is a interval subset semiring of infinite order.

We will illustrate this situation by some examples.

Example 2.14: Let M = {S([0, 20)), min, max} be the interval subset semiring. Clearly P = {subsets of the interval [0, 10)} is subsemiring.

Let W = {subsets of the interval [0, 5)} be a subsemiring of W.

B ={subsets from Z_{20}} is a subsemiring under max, min of finite order. This B is not an ideal of M, only a subset interval subsemiring.

Example 2.15: Let M = {S([0, 15)), min, max} be the subset interval semiring.

M is of infinite order.

Example 2.16: Let
S ={Collection of all subsets from [0, 9); max, min} be the subset interval semiring.

Example 2.17: Let
S = {Collection of all subsets from the interval [0, 9); max, min} be the subset interval semiring. S has subsemirings of finite order as well as infinite order.

Next we can construct using subsets of the interval [0, n) product ony pseudo interval subset ring.

DEFINITION 2.4: *Let $S = \{S([0, n)), +, \times\}$ be the set with two binary operations; $\{S([0, n), +\}$ be the group under + modulo n. $\{S([0, n)), \times\}$ be the semigroup under $\times$.*

We see $a \times (b + c) \neq a \times b + a \times c$ for all a, b, c $\in S([0, n))$.

Hence we define $S = (S([0, n)), +, \times\}$ as the interval subset pseudo ring. $o(S) = \infty$ and S is commutative.

We will first give examples of subset interval pseudo ring.

Example 2.18: Let S ={S([0, 23)), +, ×} be the subset interval pseudo ring.

Let x = [2, 4], y = (7, 10] we find x + y = [2, 4] +(7, 10]
= (9, 14] ∈ S.

Now x × y = [2, 4] × (7, 10]
= (14, 17] ∈ S.

Consider x = [2, 10], y = [5, 11] and z = [7, 20] ∈ S.

x × (y + z) = [2, 10] × ([5, 11] + [7, 20])
= [2, 10] × [12, 31]
= [2, 10] × [12, 8]
= [24, 80]
= [1, 11] I

x × y + x × z = [2, 10] × [5, 11] + [2, 10] × [7, 20]

= [10, 110] + [14, 200]
= [10, 18] + [14, 16]
= [24, 34]
= [1, 11] II

I and II are identical. Hence in this case of x, y and z satisfy the distributive law.

Now consider
$x = (3.4, 3.71)$, $y = (5.2, 1.2)$ and $z = (10.4, 2.5) \in S$.

$x \times (y + z) = (3.4, 3.71) \times ((5.2, 1.2) + (10.4, 2.5))$
$= (3.4, 3.71) \times [15.6, 3.7]$
$= (7.04, 13.727)$ I

$x \times y + x \times z =$
$(3.4, 3.71) \times (5.2, 1.2) + (3.4, 3.71) \times (10.4, 2.5)$

$= (17.68, 4.452) + (12.36, 9.275)$
$= (7.04, 13.727)$ II

I and II are identical in this case also we see $a \times (b + c) = a \times b + a \times c$.

We now give a triple where $a \times (b + c) \neq a \times b + a \times c$

Let $a = 10.5$, $b = 9.8$ and $c = 22.1 \in S$.

$a \times (b + c) = a \times (9.8 + 22.1)$
$= 10.5 \times (31.9)$
$= 10.5 \times 8.9$
$= 1.45$ I

Consider $a \times b + a \times c$
$= 10.5 \times 9.8 + 10.5 \times 22.1$
$= 10.9 + 2.05$
$= 12.95$ II

I and II are distinct we see the distributive law for this triple is not true.

Hence S is only a pseudo interval subset ring of infinite order which is commutative.

Example 2.19: Let S = {S ([0, 24)), +, ×} be the pseudo interval subset ring. S has zero divisors but only few idempotents and units.

S has however subrings of finite order as well as ideals of infinite order.

Let $P_1 = \{Z_{24}, \times, +\}$,
$P_2 = \{[0, 0], [0, n] \mid n \in Z_{24}, +, \times\}$ and
$P_3 = \{[n, 0] \mid n \in Z_{24}, \times, +\}$ be a subset interval subrings which is not pseudo.

$P_4 = \{[0, 24), +, \times\}$ is a subset interval subring which is pseudo.

This is not an ideal of S for if x = [7, 15.2] ∈ S and y = 0.03 $\in P_4$ then x × y = [7, 15.2] × 0.03 = [0.21, 4.56] ∉ P.

This is not an ideal of S.

Now we have pseudo interval subset rings built using S([0, n)).

Almost all properties enjoyed by the pseudo ring [0, n) can be adopted for the pseudo interval subset ring S([0, n)).

Now we just indicate how algebraic structures can be built usings the semigroups, under min (or max or ×), group under + the semiring under min, max and the pseudo ring using S([0, n)).

All these situations will be exhibited by examples.

Example 2.20: Let
$S = \{(a_1, a_2, a_3, a_4) \mid a_i \in S([0, 12));\ 1 \leq i \leq 4, \min\}$ be the interval subset row matrix semigroup.

Let x = ([0, 3.2], [5, 10.12], 0.74, 9.801) and y = ([0, 10.5], [3, 5.7], [2, 3.74], 8.107) $\in$ S.

min {x, y} = ([0, 3.2], [3, 5.7],0.74, 8.107) $\in$ S.

Every x $\in$ S is such that min {x, x} = x.

Thus every element in S is an idempotent.

Further if $\overline{0}$ = (0, 0, 0, 0) is the zero of S; min {x, $\overline{0}$ } = $\overline{0}$.

Example 2.21: Let

$$S = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_8 \end{bmatrix} \middle| x_i \in S([0, 11)), 1 \le i \le 8, \min \right\}$$

be the column matrix subset interval semigroup.

$$\text{Let } A = \begin{bmatrix} 0 \\ 0.71 \\ [0, 2.51] \\ [0.3, 6.5] \\ (2.01, 0.11] \\ [9, 4.3] \\ 0 \\ (0.71, 5) \end{bmatrix} \text{ and } B = \begin{bmatrix} [9.31, 10.7] \\ (0.31, 2.11) \\ 0 \\ [6.7, 4.7] \\ (1.11, 7] \\ [6.31, 2.11] \\ [7.32, 4.15] \\ [0.012, 7.2] \end{bmatrix} \in S.$$

$$\min\{A, B\} = \begin{bmatrix} 0 \\ (0.31, 0.71] \\ 0 \\ [0.3, 4.7] \\ [1.11, 0.11] \\ [6.31, 2.11] \\ 0 \\ [0.012, 5] \end{bmatrix} \in S.$$

This is the way operation is performed on S.

Example 2.22: Let

$$S = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \middle| \, a_i \in S([0, 7)), 1 \le i \le 16, \min \right\}$$

be the pseudo interval semigroup; operations are done componentwise.

If $A = (a_{ij})$ and $B = (b_{ij}) \in S$.

$\min \{A, B\} = (\min \{(a_{ij}), (b_{ij})\})$.

We will represent this by examples.

$$\text{Let } A = \begin{bmatrix} 0.75 & [0, 0.8] & [2, 6.5] & [3, 0.21] \\ 0 & [3, 2.1] & [6, 1.3] & [1, 4.01] \\ [2.3, 1.1] & 0 & [2.1, 1.1] & [4, 3.05] \\ 0 & 6.5 & 0 & 3.89 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 0.93 & 0 & [2.1, 0.11] & [2.11, 1.11] \\ [4.2, 1.1] & 0.89 & 0 & [1.11, 3.2] \\ 0 & [3, 4.21] & 1.902 & 0 \\ [0.312, 1.5] & 0 & (3.15, 2.1) & 2.17 \end{bmatrix} \in S;$$

min {A, B} =

$$\begin{bmatrix} 0.75 & 0 & [2, 0.11] & [2.11, 0.21] \\ 0 & 0.89 & 0 & [1, 3.2] \\ 0 & 0 & [1.902, 1.1] & 0 \\ 0 & 0 & 0 & 2.17 \end{bmatrix}.$$

This is the way operation is performed on S.

Example 2.23: Let

$$S = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ \vdots & \vdots & \vdots & \vdots \\ a_{37} & a_{38} & a_{39} & a_{40} \end{bmatrix} \middle| a_i \in S([0, 15)), 1 \leq i \leq 40, \min \right\}$$

be the interval subset semigroup.

Example 2.24: Let
$S = \{(a_1, a_2, a_3) \mid a_i \in S([0, 12)), 1 \leq i \leq 3, \max\}$ be the interval subset semigroup.

Let A = ([10.3, 6.3], [3.4, 5.71], [7.8, 6.5] and

B = ([11.1, 3.1], [2.1, 8.5], [3.1, 6.5]) $\in$ S.

max {A, B} = ([11.1, 6.3], [3.4,8.5], [7.8,6.5]) $\in$ S.

This is the way operation on S is performed.

Example 2.25: Let

$$S = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} \middle| a_i \in S([0, 2)), 1 \leq i \leq 6, \max \right\}$$

be the subset interval semigroup.

$$\text{Let } A = \begin{bmatrix} 0.33 \\ [1,1.5] \\ [0.7,0.51] \\ [1.35,0.65] \\ [1.31,0] \\ [0.99,1.21] \end{bmatrix} \text{ and } B = \begin{bmatrix} 0.123 \\ [0.71,0.11] \\ [0.11,1.5] \\ [1.99,0.3] \\ [1.2,0.9] \\ [1.2101,1.01] \end{bmatrix}$$

$$\max \{A, B\} = \begin{bmatrix} 0.33 \\ [1,1.5] \\ [0.7,1.5] \\ [1.99,0.65] \\ [1.31,0.9] \\ [0.2101,1.21] \end{bmatrix} \in S.$$

This is the way operations are perfomed on S.

Example 2.26: Let

$$S = \left\{\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \middle|\ a_i \in S([0, 24)), 1 \leq i \leq 9, \max\right\}$$

be the matrix interval subset semigroup.

$$\text{Let } A = \begin{bmatrix} [2.1, 5.6] & [18.3, 20.4] & [3.8, 16.7] \\ [9.8, 0.2] & 0 & [19.3, 22] \\ [19.2, 8] & [9.32, 18.7] & 19.71 \end{bmatrix}$$

$$\text{and } B = \begin{bmatrix} 2.5 & 19.5 & 0 \\ [19.6, 0] & [19.63, 8.751] & 20.1 \\ [16.7, 11.3] & [12.15, 6.37] & [21, 6.3] \end{bmatrix} \text{ be in S.}$$

$$\max\{A, B\} = \begin{bmatrix} [2.5, 5.6] & [19.5, 20.4] & [3.8, 16.7] \\ [19.6, 0.2] & [19.63, 8.751] & [20.1, 22] \\ [19.2, 11.3] & [12.15, 18.7] & [21, 19.71] \end{bmatrix}$$

is in S. This is the way operation is performed on S.

We now give examples of subset interval semigroup matrices.

Example 2.27: Let
$P = \{(a_1, a_2, a_3, a_4) \mid a_i \in S([0, 8)), 1 \leq i \leq 4, \times\}$ be the subset interval row matrix semigroup.

Let x = ([0, 3.1), (2.5, 1.7), [1.78,5.65], (7.8, 1.5)) and
y = (5, [2, 4], 6, [4, 0]) $\in$ P

$x \times y = ([0, 3.1) \times 5, (2.5, 1.7) \times [2,4], [1.78, 5.65] \times 6, (7.8, 1.5) \times [4, 0])$

$= ([0, 7.5], [5, 6.8], [2.68, 1.90], [7.2,0]) \in P.$

This is the way × operation is carred out. P has infinite number of zero divisors only finite number of idempotents and units.

Infact $P_1 = \{(a_1,0,0,0) \mid a_1 \in S([0, 8)), \times\} \subseteq P$ is a subsemigroup as well as an ideal of P.

Thus P has atleast ${}_4C_1 + {}_4C_2 + {}_4C_3$ number of subsemigroups which are ideals.

Further P has atleast $4({}_4C_1 + {}_4C_2 + {}_4C_3 + {}_4C_4)$ number of subsemigroups which are not ideals of P.

Example 2.28: Let

$$S = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \end{bmatrix} \middle| a_i \in S([0, 13)), 1 \leq i \leq 7, \times_n \right\}$$

be the subset interval column matrix semigroup.

S has atleast ${}_7C_1 + {}_7C_2 + \ldots + {}_7C_6$ number of subsemigroups which are ideals of S.

S has atleast $7({}_7C_1 + {}_7C_2 + \ldots + {}_7C_7)$ number of subsemigroups which are not ideals.

S has infinite number of zero divisors, finite number of units and idempotents.

Study of properties of S is interesting and innovative.

Example 2.29: Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \end{bmatrix} \middle| a_i \in S([0, 24)), 1 \leq i \leq 15, \times_n \right\}$$

be the interval subset matrix semigroup under natural product $\times_n$.

M has infinite number of zero divisors and finite number of units and idempotents.

$$P = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ \vdots & \vdots & \vdots \\ a_{13} & a_{14} & a_{15} \end{bmatrix} \middle| a_i \in Z_{24}, 1 \leq i \leq 15, \times_n \right\}$$

is a subsemigroup which is not an ideal.

$$P_1 = \left\{ \begin{bmatrix} a_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \middle| a_1 \in S([0, 24), \times_n \right\} \subseteq M,$$

$$P_2 = \left\{\begin{bmatrix} 0 & a_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \middle| a_2 \in S([0, 24), \times_n\} \subseteq M, \ldots,$$

$$P_{15} = \left\{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & a_{15} \end{bmatrix} \middle| a_2 \in S([0, 24), \times_n\} \subseteq M$$

are subsemigroups which are also ideals of M.

However P is finite. P_i's are of infinite order.

Example 2.30: Let

$$S = \left\{\begin{bmatrix} a_1 & a_2 & \ldots & a_8 \\ a_9 & a_{10} & \ldots & a_{16} \end{bmatrix} \middle| a_i \in S([0, 10)); 1 \leq i \leq 16, \times_n\right\}$$

be a subset semigroup of infinite order. S has infinite order.

S has infinite number of zero divisors and only finite number of units and idempotents.

Now we build subset interval matrix semiring which are only illustrated by examples.

Example 2.31: Let S = $\{(a_1, a_2, \ldots, a_{10}) \mid a_i \in S([0, 23)), 1 \leq i \leq 10, \min, \max\}$ be the interval subset row matrix semiring.

S has subsemirings and ideals.

Example 2.32: Let

$$M = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{18} \end{bmatrix} \middle| a_i \in S([0, 21));\ 1 \leq i \leq 18, \min, \max \right\}$$

be the subset interval column matrix semiring.

M has several subsemiring of both finite and infinite order. M has ideals which are only of finite order.

Example 2.33: Let

$$S = \left\{ \begin{bmatrix} a_1 & a_2 & \cdots & a_9 \\ a_{10} & a_{11} & \cdots & a_{18} \\ a_{19} & a_{20} & \cdots & a_{27} \\ a_{28} & a_{29} & \cdots & a_{36} \\ a_{37} & a_{38} & \cdots & a_{45} \end{bmatrix} \middle| a_i \in S([0, 124)); \right.$$

$$\left. 1 \leq i \leq 45, \min, \max \right\}$$

be the subset interval matrix semiring.

M has subsemirings which are not ideals of both infinite and finite order. M has infinite number of zero divisors and only finite number of idempotents.

Next we give examples of subset interval matrix pseudo rings.

Example 2.34: Let
$X = \{(a_1, a_2, a_3) \mid a_i \in S([0, 50)), 1 \leq i \leq 3, \min, \max\}$ be the subset interval row matrix semiring.

Let a = ([2.1, 4.5], [5.3, 1.5), [9.3, 12.5)) and

b = ([10, 20], [40, 30], [20, 1]) $\in$ X;

min {a, b} = {([2.1, 4.5], [5.3, 1.5], [9.3, 1])}

and max {a, b} ={([10, 20], [40, 30], [20,12.5])} $\in$ X.

Now if we define $\times$ and + on X we see the + and $\times$ do not distribute that is why we call them as pseudo ring.

a + b = ([12.1, 24.5], [45.3, 31.5], [29.3, 13.5])

and a $\times$ b = {[21, 40], [12, 45], [36, 12.5]) $\in$ X.

This is the way operations are performed and are different, X has infinite number of zero divisors.

X has units, (1, 1, 1) is the unit element of X under product. X has only finite number of units and idempotents.

$P_1 = \{(a_1, 0, 0) \mid a_1 \in S([0, 50), +, \times\} \subseteq X$ is a pseudo subring as well as pseudo ideal of X.

$P_2 = \{(0, a_2, 0) \mid a_2 \in S([0,50)), +, \times\} \subseteq X$ is a pseudo row matrix subring as well as pseudo ideal of X.

$P_3 = \{(0, 0, a_3) \mid a_3 \in S([0, 50)), +, \times\} \subseteq X$ is a pseudo row matrix ideal of X.

$P_{1,2} = \{(a_1, a_2, 0) \mid a_1, a_2 \in S([0, 50)), +, \times\} \subseteq X$ is a pseudo row matrix ideal of X.

X has atleast ${}_3C_1 + {}_3C_2$ number of pseudo ideals.

Example 2.35: Let

$$W = \left\{\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_9 \end{bmatrix} \middle| a_i \in S([0, 43)); 1 \leq i \leq 9, +, \times_n\right\}$$

be the pseudo interval subset column matrix ring.

W has atleast ${}_9C_1 + {}_9C_2 + \ldots + {}_9C_8$ number of pseudo ideals W also has atleast 5 $({}_9C_1 + {}_9C_2 + \ldots + {}_9C_9)$ number of pseudo subrings which are not ideals of W.

Example 2.36: Let

$$V = \left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & \ldots & \ldots & a_8 \\ a_9 & \ldots & \ldots & a_{12} \\ \vdots & & & \vdots \\ a_{37} & \ldots & \ldots & a_{40} \end{bmatrix} \middle| a_i \in S([0, 48)); 1 \leq i \leq 40, +, \times_n\right\}$$

be the subset interval matrix pseudo ring.

V has infinite number of zero divisors. Finite number of units and idempotents.

Now we can in all other cases of semigroups, semirings and pseudo rings replace the intervals S([0,n)) by the collection of all of natural class of intervals (S(C([0, n)) or by S(⟨[0, n)∪ I⟩) or by S(C⟨[0, n)∪ I⟩) and study those algebraic structures.

This work is considered as a matter of routine.When S([0,n)) is replaced by S(C([0, n)) we call those algebraic structures as subset interval finite complex modulo integer semiring or semigroup or pseudo ring.

When S([0, n)) is replaced by S(⟨[0, n)∪ I⟩) we call those algebraic structures as interval subset neutrosophic semigroup (semiring or pseudo ring).

Finally if S([0, n)) is replaed by S(C⟨[0, n)∪ I⟩)) we call those algebraic structures as subset interval finite complex modulo integer neutrosophic semigroup (or semiring or pseudo ring).

Finally if S([0, n)) is replaced by S(C⟨[0, n)∪ I⟩)) we call those algebraic structures as subset interval finite complex modulo integer neutrosophic semigroup (or semiring or pseudo ring).

We will illustrate these situation by few examples.

As our main motivation in this book is to study topological spaces using subsets from [0, n) and S([0, n)).

Example 2.37: Let P = {S(C([0, 10))) = {Collection of all intervals subsets in C([0,10))}, min} be a subset interval finite complex modulo integer semigroup.

$[3, 2i_F] = [3 + 0i_F, 0 + 2i_F]$ and so on. $[3i_F, 2 + 5i_F]$ is again $[0 + 3i_F, 2 + 5i_F]$.

We find min $\{a + i_Fb, c + di_F\}$; min $\{a, c\}$ + min $\{di_F, bi_F\}$ and so on.

Let $x = [0.3 + 0.8i_F, 9 + 6i_F]$ and
$y = [0.15 + 7i_F, 4 + 7i_F] \in P$.

min $\{x, y\} = [0.15 + 0.8i_F, 4 + 6i_F]$.

This is the way min operation is performed on P.

It is easily verified P is an infinite subset semiring. P is of infinite order.

Every singleton is a subset interval subsemiring of order one.

Further a two element subset of P in general need not be a subset interval subsemigroup.

For p = $\{[0.3i_F, 5]\}$ and q = $\{[2i_F + 4, 2]\} \in P$

min $\{p,q\} = \{0.3i_F, 2\} \notin P$.

S = {p, q} is only a subset of P but is not a subsemigroup of P. We see $S_c = \{p, q, [0.3i_F, 2]\} \subseteq P$ is a subset interval subsemigroup.

We see $B = \{x = [0, 8i_F], [2 + 5i_F, 8 + 4i_F] = y, z = [2 + 3i_F, 9i_F]\} \subseteq P$ is a subset of P.

min $\{x, y\}$ = min $\{[0, 8i_F], [2 + 5i_F, 8 + 4i_F]\}$
$= [0, 4i_F]$; min $\{y, z\} = [2 + 3i_F, 4i_F]$;
min $\{x, z\} = [0, 8i_F] = x$.

Thus $B_C = \{x, y, z, [0, 4i_F], [2 + 3i_F, 4i_F]\} \subseteq P$ is a subset interval subsemigroup.

We can always complete the subset of P into a subsemigroup under min operation.

Example 2.38: Let M = {S(C[0, 17)), min} be the subset interval semigroup. M has subsemigroups ideals and idempotents.

Example 2.39: Let P= $\{S(C\langle[0, 13) \cup I\rangle), \min\}$ be the subset interval neutrosophic semigroup.

Let x = [6.3 + 2I, 7 + 5I] and y = [3.7 + 4I, 3 + 10I] $\in$ P,

min{x, y} = [3.7 + 2I, 3 + 5I] $\notin$ {x, y}.

So the completed subsemigroup is {x, y, [3.7 + 2I, 3 + 5I]}.

Example 2.40: Let T $=\{S(\langle[0, 125) \cup I\rangle), \min\}$ be the subset interval neutrosophic semigroup.

Let $x = [6.3 + 28I, 3 + 120I]$ and
$y = [0.7 + 110I, 4.5 + 8I] \in T$,

$\min \{x, y\} = [0.7 + 28I, 3 + 8I] \notin \{x, y\}$.

Thus the subset $\{x, y\}$ can be completed into a subsemigroup.

Example 2.41: Let P = $S(C\langle[0, 53) \cup I\rangle)), \min\}$ be the subset interval neutrosophic semigroup. P has several subsemigroups of order one, two and so on.

Example 2.42: Let M = $S(C\langle[0, 41) \cup I\rangle)), \min\}$ be the subset interval neutrosophic complex modulo integer semigroup. M has all properties of an interval subset semigroup.

Study in this direction is also a matter of routine and interested reader can derive almost all results.

Let $x = [3 + 2.1i_F + 9.10I + 21i_FI, 4 + 2I + 3.4i_F + 10i_F I]$ and
$y = [2 + 8i_F + 3I + 10i_FI, 7 + I + 0.2i_F + 7i_FI] \in M$.

$\min \{x, y\} = [2 + 2.1i_F + 3I + 10i_FI, 4 + I + 0.2i_F + 7i_FI] \notin \{x, y\}$. Thus $\{x, y, \min \{x, y\}\}$ is a subsemigroup.

Now if the min operation is replaced by the max operation we get interval subset semigroups and one can as a matter routine derive all the properties.

Here we describe this by one or two examples.

Example 2.43: Let S = $\{S(C([0, 20)), \max\}$ be the subset interval semigroup.

Let $x = [3i_F + 12, 4.5i_F + 8]$ and
$y = [12i_F + 7, 2.5i_F + 15] \in S$

$\max \{x, y\} = [12i_F + 12, 4.5i_F + 15] \notin \{x, y\}$.

Thus the subset $\{x, y\}$ can be completed to form a subset interval subsemigroup.

Example 2.44: Let $M = \{S(C\langle[0, 7) \cup I\rangle)); \max\}$ be the subset interval semigroup. Every singleton element is subset interval subsemigroup.

Example 2.45: Let
$A = \{(a_1, a_2, a_3, a_4) \mid a_i \in S(C([0,10)); 1 \leq i \leq 4, \max\}$ be the subset interval semigroup of row matrix.

All properties can be derived.

Example 2.46: Let

$$A = \left\{ \left[\begin{array}{c} a_1 \\ a_2 \\ \vdots \\ a_9 \end{array} \right] \middle| a_i \in S(C[0, 12)); 1 \leq i \leq 9, \min \right\}$$

be the subset interval semigroup of column matrix. A has subsemigroups of all orders say one, two, three and so on. A has ideals.

Infact A has atleast ${}_9C_1 + {}_9C_2 + \ldots + {}_9C_8$ number of ideals.

A has infinite number of subsemigroups.

Example 2.47: Let

$$M = \left\{ \left. \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ \vdots & \vdots & \vdots & \vdots \\ a_{33} & a_{34} & a_{35} & a_{36} \end{bmatrix} \right| a_i \in S(C\langle[0, 24) \cup I\rangle); \right.$$

$$1 \leq i \leq 36, \ \min\}$$

be the interval neutrosophic semigroup of matrices.

M has infinite number of subsemigroups.

M has atleast ${}_{36}C_1 + {}_{36}C_2 + \ldots + {}_{36}C_{35}$ number of ideals.

Example 2.48: Let

$$M = \left\{ \left. \begin{bmatrix} a_1 & a_2 & \ldots & a_8 \\ a_5 & a_6 & \ldots & a_{16} \\ a_{17} & a_{18} & \ldots & a_{24} \\ a_{25} & a_{26} & \ldots & a_{32} \end{bmatrix} \right| a_i \in S(C\langle[0, 28) \cup I\rangle); \right.$$

$$1 \leq i \leq 32, \ \min\}$$

be the subset interval finite complex neutrosophic semigroup of matrices.

M has astleast ${}_{32}C_1 + {}_{32}C_2 + {}_{32}C_{31}$ number of ideals and $7({}_{32}C_1 + {}_{32}C_2 + \ldots + {}_{32}C_{32})$ number of subsemigroups which are not ideals apart from the subsemigroups of order one, order two and so on.

Example 2.49: Let

$$V = \left\{\left[\begin{array}{ccc} a_1 & a_2 & a_3 \\ \hline a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ \hline a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} \\ \hline a_{19} & a_{20} & a_{21} \\ a_{22} & a_{23} & a_{24} \end{array}\right] \middle| \; a_i \in S(C\langle[0, 24) \cup I\rangle);\ 1 \leq i \leq 24,\ \max\right\}$$

be a interval subset super matrix neutrosophic semigroup.

Next we give a few examples of subset interval semigroups under $\times$ using $S(C([0, n))$, $S(C\langle[0, n) \cup I\rangle)$ and $S(\langle[0, n) \cup I\rangle)$.

Example 2.50: Let
$S = \{S(C([0, 21)) =$ All intervals from $C([0, 21), \times\}$ be the interval subset complex finite modulo integer semigroup. S has infinite number of zero divisors.

Let $a = [2.5 + 3.7i_F, 5 + 0.8i_F]$ and
$b = [6 + 8i_F, 9 + 4i_F] \in S$.

$a \times b = [19 + 0.2i_F, 4 + 6.2i_F] \in S$.

This is the way product operation is performed.
$x = 5.2i_F$ and $y = [7 + 5i_F, 2 + 10i_F] \in S$.

$x \times y = 5.2i_F \times [7 + 5i_F, 2 + 10i_F]$
$= [36.4i_F + 26 \times 20, 10.41i_F + 52 \times 20]$
$= [15.4i_F + 16, 10.4i_F + 11] \in S$.

S has finite subsemigroups which are not ideals.
S has infinite subsemigroups which are not ideals.
S has infinite subsemigroups which are ideals.

Example 2.51: Let

$$A = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_9 \end{bmatrix} \middle| a_i \in S(C\langle[0, 12)\rangle); 1 \leq i \leq 9, \times_n \right\}$$

be the subset interval neutrosophic semigroup of column matrices.

M has infinite number of zero divisors and atleast $4({}_9C_1 + {}_9C_2 + \ldots + {}_9C_8)$ number of ideals.

Example 2.52: Let V = $\{S(\langle[0, 43) \cup I\rangle), \times\}$ be the subset interval neutrosophic semigroup. V has units, zero divisors and idempotents. All ideals in S are of infinite order.

Example 2.53: Let
M = $\{(a_1, a_2, a_3, a_4) \mid a_i \in S(C([0, 15)), 1 \leq i \leq 4, \times\}$ be the subset interval finite complex modulo integer semigroup. M has infinite number of zero divisors. M also has subsemigroups of finite order none of which are ideals.

Example 2.54: Let V = $\{S(\langle[0, 28) \cup I\rangle), \times\}$ be the neutrosophic interval subset semigroup. V has zero divisors, units and idempotents.

$P_1 = \{Z_{28}, \times\}$ is a subsemigroup of order 28.

$P_2 = \{\langle Z_{28} \cup I\rangle, \times\}$ is a subsemigroup of finite order.

$P_3 = \{[0, 28), \times\}$ is the subsemigroup of infinite order.

$P_4 = \{\langle[0, 28) \cup I\rangle, \times\}$ is a subsemigroup of infinite order.

$M_1 = \{[0, aI] \mid a \in [0, 28), \times\}$ is a subsemigroup which is also an ideal of V.

Example 2.55: Let $T = \{S(\langle C[0, 24) \cup I\rangle)), \times\}$ be the subset interval finite complex modulo integer semigroup.

T has subsemigroups of finite and infinite order.

Example 2.56: Let

$$M = \left\{\left[\begin{array}{c} a_1 \\ a_2 \\ \vdots \\ a_{18} \end{array}\right] \middle| \, a_i \in S(C\langle[0, 23) \cup I\rangle); 1 \leq i \leq 18, \times_n\right\}$$

be the subset interval finite complex neutrosophic modulo integer column matrix semigroup.

M has infinite number of zero divisors, finite number of units and idempotents.

M has ideals all of which are of infinite order. M has finite subsemigroups which are not ideals.

Example 2.57: Let

$$W = \left\{\left[\begin{array}{ccc} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{array}\right] \middle| \, a_i \in S(C\langle[0, 23) \cup I\rangle); 1 \leq i \leq 9, \times_n\right\}$$

be the subset interval neutrosophic semigroup. W has infinite number of zero divisors.

W has atleast $3[{}_9C_1 + {}_9C_2 + \ldots + {}_9C_8]$ number of subset interval neutrosophic ideals.

W has only finite number of units and idempotents.

Example 2.58: Let

$$M = \left\{ \left[\begin{array}{cc|cc} a_1 & a_2 & a_3 & a_4 \\ a_5 & \dots & \dots & a_8 \\ \hline a_9 & \dots & \dots & a_{12} \\ a_{13} & \dots & \dots & a_{16} \\ a_{17} & \dots & \dots & a_{20} \\ \hline a_{21} & \dots & \dots & a_{24} \end{array} \right] \middle| \; a_i \in S(C\langle[0, 9) \cup I\rangle); \right.$$

$$1 \leq i \leq 24, \times_n \}$$

be the subset interval neutrosophic super matrix semigroup.

M has infinite number of zero divisors only finite number of units and idempotents.

$$e = \left[\begin{array}{cc|cc} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 \end{array} \right] \text{ is the unit of M.}$$

$$\text{M has } P_1 = \left[\begin{array}{cc|cc} 1 & 1 & 1 & 1 \\ I & I & 0 & 1 \\ \hline 0 & 0 & 1 & I \\ 1 & 1 & 0 & I \\ 0 & 0 & 0 & 0 \\ \hline 1 & 1 & I & I \\ I & I & 1 & 1 \end{array} \right] \text{ to be an idempotent.}$$

Likewise we have interval subset pseudo groups using S(C(([0, n)), S(⟨[0, n) ∪ I⟩) and S(C⟨[0, n) ∪ I⟩).

Example 2.59: Let {S(C([0,19)), +}= {Collection of all intervals from C([0, 19)), +} = G be the subset interval finite complex modulo integer pseudo group.

Let $x = [3 + 2i_F, 5 + i_F]$ and
$y = [12 + 7i_F, 15 + 10i_F] \in G$

$x + y = [15 + 9i_F, 1 + 11i_F] \in G.$

0 is the additive identity of G.

If $x = [3i_F,5]$ and $y = [7.2, i_F] \in G$ then

$x + y = [7.2 + 3i_F, 5 + i_F] \in G.$

If $x = [12.75 + 3.8i_F, 10.37 + 9i_F] \in G$ then

$–x = [6.25 + 15.2i_F, 8.63 + 10i_F] \in G$ is such that
$x + (–x) = [0, 0]$

Thus every $x \in G$ may have an inverse in G.

Example 2.60: Let

$$M = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{27} \end{bmatrix} \middle| \; a_i \in S([0, 28)); + \right\}$$

be the subset interval finite complex modulo integer pseudo group.

M is a commutative group of infinite order. H has subgroups of both finite and infinite order.

Example 2.61: Let P = S(C⟨[0, 12) ∪ I⟩), +} be the subset interval neutrosophic pseudo group of infinite order.

Let x = [9.31 + 3I, 6.7 + 10.3I]
and y = [2.79 + 4.23I] ∈ P.

x + y = [0.1 + 7.23I, 9.49 + 2.53I] ∈ P.

This is the way + operation is performed on P.

Example 2.62: Let

$$V = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{19} & a_{20} & a_{21} \end{bmatrix} \middle| a_i \in S(C\langle[0, 24) \cup I\rangle); 1 \leq i \leq 21, + \right\}$$

be the interval subset neutrosophic pseudo group V. V has several subgroups of both finite and infinite order.

Example 2.63: Let W = {S(⟨[0, 31) ∪ I⟩), +} be the interval subset neutrosophic pseudo group. W has finite subgroups as well as infinite pseudo subgroups.

Example 2.64: Let W = {S(C⟨[0, 24) ∪ I⟩), +} be the interval subset finite complex modulo integer pseudo subset group. S = {Collection of all intervals from the (C⟨[0, 24) ∪ I⟩)}.

Let x = [3.2 + 10.5i_F + 4.8I + 20i_FI, 16.3 + 5.4i_F + 3.2I + 16i_FI]

and y = 8+ 4i_F + 5I + 6i_FI ∈ S. Clearly x + y ∈ W.

Consider x × y = $[25.6 + 84i_F + 14.4I + 160i_FI + 12.8i_F + 42i_F^2 + 19.2Ii_F + 80i_F^2I + 16.0I + 52.5i_FI + 24.0I + 100i_FI + 19.2i_FI + 63i_F^2I + 28.8i_FI + 120i_F^2I,\ 130.4 + 43.2i_F + 25.6I + 128i_FI + 65.2i_F + 21.6i_F^2 + 12.8Ii_F + 64Ii_F^2 + 81.5I + 27.0i_F + 16.0I + 80Ii_F + 97.8i_F^2I + 32.4i_FI + 19.2Ii_F + 96i_F^2I]$

Simplify using $i_F^2 = 23$, $i_F^2 I^2 = 23I \bmod 24$.

This is the way product is performed on S. This operation product, × will be used when pseudo linear algebra is to be defined.

S has infinite number of zero divisors, a few units and idempotents. However (S, +) is a pseudo group.

Example 2.65: Let

$$W = \left\{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{18} \end{bmatrix} \middle|\ a_i \in \{S(C\langle[0, 17) \cup I\rangle);\ 1 \le i \le 18, +\}\right.$$

be the finite complex modulo integer neutrosophic interval subset pseudo group.

Example 2.66: Let B = $\{S(C\langle[0, 29) \cup I\rangle), +\}$ be the subset interval finite complex modulo integer neutrosophic pseudo group of infinite order.

We will be using this concept to build subset interval pseudo vector spaces and subset interval pseudo linear algebras.

Example 2.67: Let

$$B = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \end{bmatrix} \middle| \, a_i \in S(C\langle[0, 23) \cup I\rangle); 1 \le i \le 12, + \right\}$$

be the interval subset finite complex modulo integer neutrosophic matrix pseudo group.

Next we proceed onto give examples of them.

Example 2.68: Let V = S(C(0, 23)), +} be the pseudo vector space of interval subsets of finite complex number over the field Z_{23}. V has pseudo subspaces V of infinite dimension. V has pseudo subspaces of finite dimension also.

$S_1 = \{Z_{23}, +\}$ is a pseudo vector subspace of V of dimension 1.

$S_2 = \{[0, a] \mid a \in Z_{23}, +\}$ is again a pseudo vector subspace of V and so on.

Example 2.69: Let V = $S(C(\langle[0, 29) \cup I\rangle), +\}$ be the pseudo vector space of subset interval complex neutrosophic integers over the field Z_{29}. V has subset interval pseudo subspaces of both finite and infinite dimension over Z_{29}.

Example 2.70: Let
$W = \{(a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mid a_i \in S(([0, 7)), 1 \le i \le 7, +\}$ be the subset interval pseudo vector space over the field Z_7. W has several subspaces of both finite and infinite dimension over Z_7.

$V_1 = \{(a_1, 0, \ldots, 0) \mid a_1 \in S([0,7)), +\} \subseteq W$,
$V_2 = \{(0, a_2, 0, \ldots, 0) \mid a_2 \in S([0,7)), +\} \subseteq W, \ldots,$

$V_7 = \{(0, 0, \ldots, 0, a_7) \mid a_7 \in S([0,7)), +\}$ are subset interval pseudo subspaces of W such that $V_i \cap V_j = \{(0, 0, \ldots, 0)\}$ if $i \neq j$, $1 \leq i, j \leq 7$.

Further $W = V_1 + V_2 + \ldots + V_7$.

Infact W has atleast ${}_7C_1 + {}_7C_2 + \ldots + {}_7C_6$ number of pseudo subspaces of infinite dimension over Z_7.

W has atleast $4({}_7C_1 + {}_7C_2 + \ldots + {}_7C_7)$ number of pseudo subspaces of finite dimension over Z_7.

Example 2.71: Let

$$M = \left\{\left.\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}\right| a_i \in S(([0, 9)), 1 \leq i \leq 9, +\right\}$$

be interval subset pseudo vector space over the field Z_{19}. M is a interval subset finite complex modulo integer pseudo vector space over Z_{19}.

Example 2.72: Let $W = \{S(C([0, 29)), +\}$ be a subset interval pseudo space over the field Z_{29}. If product is defined on W; W becomes a pseudo linear algebra over Z_{29}. However W is infinite dimension over Z_{29}.

Example 2.73: Let

$$M = \left\{\left.\begin{bmatrix} a_1 & a_2 & \ldots & a_9 \\ a_{10} & a_{11} & \ldots & a_{18} \\ a_{19} & a_{20} & \ldots & a_{27} \\ a_{28} & a_{29} & \ldots & a_{36} \\ a_{37} & a_{38} & \ldots & a_{45} \end{bmatrix}\right| a_i \in S(([0, 53)), 1 \leq i \leq 45, +\right\}$$

be a subset interval pseudo vector space over the field Z_{53}.

However using the natural product $\times_n$; S is a subset interval pseudo linear algebra over Z_{53}.

Example 2.74: Let M = {S(⟨[0, 61) ∪ I⟩), +} be the subset interval neutrosophic pseudo vector space over the field Z_{61}. M is only a subset interval neutrosophic pseudo linear algebra over Z_{61}.

Example 2.75: Let

$$V = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \\ a_{11} & a_{12} \end{bmatrix} \middle| \, a_i \in S(\langle[0, 43) \cup I\rangle), 1 \leq i \leq 12, +, \times_n \right\}$$

be the subset interval neutrosophic pseudo linear algebra over the field Z_{43}.

Dimension of V over Z_{43} is infinite. V has atleast $9({}_{12}C_1 + {}_{12}C_2 + \ldots + {}_{12}C_{11}) + 8$ number of linear algebras some of them are finite dimensional and some are pseudo and some are not pseudo.

It is interesting but a matter of routine to study them.

Example 2.76: Let

$$V = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ \vdots & \vdots & \vdots \\ a_{58} & a_{59} & a_{60} \end{bmatrix} \middle| \, a_i \in S(\langle[0, 23) \cup I\rangle), 1 \leq i \leq 60, +, \times_n \right\}$$

be the pseudo subset interval linear algebra over the field Z_{23}. Dimension of W over Z_{23} is infinite.

W has atleast ${}_{60}C_1 + {}_{60}C_2 + \ldots + {}_{60}C_{59}$ number of pseudo subspaces of infinite dimension.

W has atleast $4({}_{60}C_1 + {}_{60}C_2 + \ldots + {}_{60}C_{60})$ number of finite dimension vector subspaces which are not pseudo.

Example 2.77: Let

$$S = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \middle| \; a_i \in S(\langle [0, 26) \cup I \rangle), \right.$$

$$\left. 1 \leq i \leq 16, +, \times \right\}$$

be the non commutative pseudo interval subset linear algebra of finite complex modulo integer neutrosophic matrices. S is of infinite dimension has both subspaces of finite and infinite dimension.

We can build vector spaces over pseudo S-rings like [0, n) or C([0, n) or $\langle [0, n) \cup I \rangle$ or $C(Z_n)$ or $\langle Z_n \cup I \rangle$ or $C(\langle Z_n \cup I \rangle)$ or $C\langle [0, n) \cup I \rangle$) or S([0, n)) or S(C([0, n)) or $S(\langle [0, n) \cup I \rangle)$ or $S(C(\langle [0, n) \cup I \rangle)$.

When they are built over S([0, n)) or S(C([0, n)) or $S(\langle [0, n) \cup I \rangle)$ or $S(C\langle [0, n) \cup I \rangle)$ we call them as strong pseudo interval subset linear algebra or strong subset interval vector space.

Only in case of strong interval subset pseudo linear algebras, we will be in a position to built inner product or linear functionals.

All these work is considered as a matter of routine and left as an exercise for the reader.

However to make the later chapters on pseudo topological interval spaces, some examples are presented.

Example 2.78: Let

$$S = \left\{ \begin{bmatrix} a_1 & a_2 & \ldots & a_9 \\ a_{10} & a_{11} & \ldots & a_{18} \\ a_{19} & a_{20} & \ldots & a_{27} \end{bmatrix} \middle| \; a_i \in S(\langle[0, 23) \cup I\rangle), 1 \leq i \leq 27, + \right\}.$$

S is a S-strong interval subset vector space over the subset interval pseudo neutrosophic ring R = $\{S(\langle[0, 23) \cup I\rangle), +, \times\}$.

S is a S-interval subset pseudo vector space over the subset interval pseudo ring B = $\{S([0, 23)), +, \times\}$.

Let S be a S-interval subset pseudo vector space over the pseudo interval neutrosophic S-ring B = $\{S(\langle[0, 23) \cup I\rangle), +, \times\}$.

Thus S is a S-interval vector space over several S-pseudo rings but is a strong Smarandache pseudo vector space only over the S-subset interval pseudo ring. $\{S(\langle[0, 23) \cup I\rangle), +, \times\}$.

Example 2.79: Let V = $\{(a_1, a_2, a_3) \mid a_i \in S([0, 23)), 1 \leq i \leq 3, +, \times\}$ be the Smarandahe strong interval subset pseudo vector space over the S-subset interval pseudo ring S ([0, 23)).

V has subspaces.

V is a inner product space.

If $x = (x_1, x_2, x_3)$ and $y = (y_1, y_2, y_3) \in V$;

$$\langle x, y\rangle = \sum_{i=1}^{3} x_i y_i \pmod{23}.$$

Let x = ([0, 2.3], [7.3, 2.5], 9.1) and
y = (9, [8, 4), (10, 5)) $\in$ V.

$\langle x, y\rangle$ = [0, 2.3] × 9 + [7.3, 2.5] × [8, 4] + 9.1 × (10, 5))

= [0, 20.7] + [58.4, 14.0] + [91, 45.5] (mod 23)

= [11.4, 11.2] $\in$ V.

This is the way inner product is performed on V.

Thus V becomes an inner product interval S-strong pseudo vector space.

Example 2.80: Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \middle| \; a_i \in S(C([0, 13))),\ 1 \le i \le 9, +, \times_n \right\}$$

be the strong Smaradache subset interval pseudo linear algebra of finite complex modulo integers.

M is a inner product space.

$$f : M \to S(C([0, 13)) \text{ is such that}$$

$$f(A) = \sum_{i=1}^{9} a_i \text{ (mod 13); } A \in M;$$

f is a linear functional on M.

Example 2.81: Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} \end{bmatrix} \middle| \; a_i \in S(\langle [0, 3) \cup I \rangle), 1 \leq i \leq 18, +, \times_n \right\}$$

be the Smaradache strong subset interval pseudo linear algebra over the S-subset interval neutrosophic pseudo ring.

P = $\{S(\langle [0, 3) \cup I \rangle), +, \times\}$. M has several S-subset interval neutrosophic pseudo subspaces and none of them are of finite cardinality.

Example 2.82: Let

$$S = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \end{bmatrix} \middle| \; a_i \in S(C\langle [0, 19) \cup I \rangle), 1 \leq i \leq 7, +, \times_n \right\}$$

be the Strong Smaradache subset interval finite complex modulo integer pseudo linear algebra of column matrices over the S-subset interval finite complex modulo integer pseudo ring $S(C\langle [0, 19) \cup I \rangle), +, \times\}$.

Example 2.83: Let

$$S = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \\ a_{11} & a_{12} \end{bmatrix} \middle| \; a_i \in S(C\langle[0, 23)\rangle), 1 \leq i \leq 12, +, \times_n \right\}$$

be the S-strong subset interval finite complex modulo integer matrix pseudo linear algebra over the S-subset interval finite complex modulo integer pseudo ring R = {S(C([0, 23)), +, ×}.

S is a inner product S-strong pseudo space.

Several linear functional can be defined on S.

We can also define linear operators on S.

Let T : S → S be defined.

$$T \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \\ a_{11} & a_{12} \end{bmatrix} \right\} = \begin{bmatrix} a_1 & 0 \\ 0 & a_4 \\ a_5 & 0 \\ 0 & a_7 \\ a_8 & 0 \\ 0 & a_9 \end{bmatrix}.$$

It is easily verified T is a linear operator on S.

Example 2.84: Let

$$W = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & \cdots & \cdots & a_8 \\ \vdots & \vdots & \vdots & \vdots \\ a_{61} & a_{62} & a_{63} & a_{64} \end{bmatrix} \middle| \; a_i \in S(C\langle[0, 29) \cup I\rangle), \; 1 \le i \le 64, +, \times \right\}$$

be the S-strong subset interval neutrosophic matrix pseudo linear algebra over the S-subset interval neutrosophic pseudo ring. $R = S(C\langle[0, 29) \cup I\rangle), +, \times\}$.

Now we define super subset interval and super strong subset interval of [0, n).

SS([0, n) = {all subsets of [0, n)}.

Clearly if $x \in [0, n)$, it is only a singleton not an interval.

We will first illustrate this situation by an example.

Let SS([0, 3)) $= \{\{0\}, \{x\}, x \in [0, 3), \{x, y\} \; (x \neq y), x, y \in [0, 3) \; \{x, y, z\} = (x \neq y, y \neq z, \text{and } x \neq z), x, y, z \in [0, 3)$ and so on}.

Thus SS([0, 3)) contains a P such that P is of infinite order.

To be more concrete take

$P_1 = \{0, 0.3, 0.752, 1.38, 1.1, 2.14, 1.441, 0.105, 0.0132\}$ is an element of SS([0, 3)).

$P_2 = \{0.001, 1.005, 0.478921\}$ is an element of SS([0, 3)). Here $P_1 \cap P_2 = \phi$.

Now suppose we define on SS([0, 3) '$\cup$', union operation then, we see under the operation '$\cup$', union SS([0, 3)) is closed.

Similarly under '$\cap$' SS([0,3)) is closed provided we agree to induct or include in SS([0, 3)) the empty set ϕ.

Thus {SS([0, 3)), $\cup$} is a semilattice or an idempotent semigroup.

Likewise {SS([0, 3)) $\cup$ ϕ, $\cap$} is a semigroup.

Hence {(SS([0, 3)) $\cup$ {ϕ}, $\cup$, $\cap$} is a semiring.

Now we can define this situation.

Let SS([0, n)) = {Collection of all subsets from the interval [0, n)}.

We see if P and Q are two subsets of SS([0, n)) then we can define P $\cap$ Q and P $\cup$ Q. It may so happen that P $\cap$ Q may be empty so we adjoin the empty set ϕ with SS([0, n)).

Further {SS([0, n)) $\cup$ {ϕ}, $\cup$, $\cap$} is a semiring.

We will first illustrate this with examples.

Example 2.85: Let {SS([0,10)), $\cup$} be the super subset interval semigroup.

Let x = {0.53, 9.2, 3.001, 8.1031} and
y = {9.2, 1.632, 3.001, 4.52, 5.3011, 0.0014} $\in$ SS([0, 10)).

x $\cup$ y = {0.53, 9.2, 3.001, 8.1031, 1.632, 4.52,
5.3011, 0.0014} $\in$ SS([0, 10)).

This is the way operation is performed on SS([0, 10)).

Clearly x $\cup$ x = x and y $\cup$ y = x.

Every singleton set is a subsemigroup.
We see x is a subsemigroup. y is a subsemigroup.

However P = {x, y} is not a subsemigroup for $x \cup y \notin P$ but P_C = {x, y, $x \cup y$} is a subsemigroup which is the completion of the subset P.

Every subset of P can be completed to form a subsemigroup.

Let A = {3, 2, 5.1, 7.5},
B = {4, 3, 6.5, 1} and
C = {2, 1, 7.312, 8, 9.1} ∈ SS([0, 10)).

Clearly M = {A, B, C} is not a subsemigroup.

For $A \cup B$ = {3, 2, 5.1, 7.5, 4, 6.5, 1} ∉ M.

$A \cup C$ = {2.1, 5.1, 7.5, 8, 7.312, 9.1} ∉ M

$B \cup C$ = {4, 3, 6.5, 1, 2, 7.312, 8, 9.1} ∉ M.

But {A, B, C, $B \cup C$, $A \cup C$, $A \cup B$} = M_C is the completion of the set M and M_C is the subsemigroup of the completed set M.

Example 2.86: Let B = {SS([0, 5)), ∪} be the super subset interval semigroup. B is of infinite order. Every element is a subsemigroup under ∪.

Consider P = {2.156} and Q = {3, 1.005} ∈ B.

$P \cup Q$ = {3, 1.005, 2.156} ≠ P or Q.

Let {P, Q} = R is not a subsemigroup however {P, Q, $P \cup Q$} is a subsemigroup called the completion of R denoted by R_C.

Let x = {0.3, 0.251, 0.001}
y ={1.21, 2.36, 3.1114} and z = {0.3, 2.36, 1.247} ∈ B.

Let T = {x, y, z} $\subseteq$ B.

Clearly T is not a subsemigroup only a subset of B.

However T_C = {x, y, z, x $\cup$ y, x $\cup$ z, z $\cup$ y} $\subseteq$ B is a subsemigroup of B called the completed subsemigroup of the subset T.

Next we give examples of super subset interval semigroup under intersection.

Example 2.87: Let {SS([0, 12)) $\cup$ {ϕ}, $\cap$} = S be the super subsemigroup interval semigroup.

Every element in S is an idempotent. Every element in S is a subsemigroup.

Let x = {10.32, 5.76} and y = {5.76, 3.21, 4} $\in$ S,

x $\cap$ y = {5.76}.

Clealry T = {x, y} is ony a subset and not a subsemigroup of S.

T_C the completion of the subset T is a subsemigroup given by {x, y, x $\cap$ y} of order three.

Let x = {4}, y = {2} and z = {7} $\in$ S.

P = {x, y, z} $\subseteq$ S, P is only a subset. But P $\cup$ {ϕ} = P_C is a subsemigroup of S.

Thus if we have n sets $x_1, \ldots, x_n$ such that $x_i \cap x_j = \phi$ if $i \neq j$; $1 \leq i, j \leq n$ then x = {$x_1, x_2, \ldots, x_n, \phi$} $\subseteq$ S is a subsemigroup.

Always every subset of S can be completed to form a subsemigroup.

Example 2.88: Let W = {SS([0, 23)) $\cup$ {ϕ}, $\cap$} be the super subset interval semigroup. o(W) = ∞. W has infinite number of subsemigroups. All subsets of order one are subsemigroups of W.

Every pair of elements in W are not in general subsemigroups. Those which are not subsemigroups only subsets can be completed to be a subsemigroups of W. Likewise every subset of W can be completed to a subsemigroup.

Let x = {0.31, 2.11, 8.131, 0, 4.28} and
y = {7.01, 2.11, 0, 5.31, 4.28} $\in$ W.

x $\cap$ y = {0, 2.11, 4.28} $\in$ W. So T = {x, y, x $\cap$ y} $\subseteq$ W is a subsemigroup of W.

Example 2.89: Let {SS([0, 21)), min} = S be the super subset interval semigroup. Let x = {3.15} and y = {2.12, 8.7} $\in$ S.

min {x, y} = {2.12, 3.15}; so P = {x, y, min {x, y}} is a subsemigroup.

Let x = {0.213, 19.32, 6.11} and y = {0.532, 10.34, 4, 8.39} $\in$ S.

min {x, y} = {0.213, 0.532, 6.11,10.34, 4, 8.39}.

Let x = {0.39, 1.23, 10.42} and
y = {0.2, 1.23, 6.5, 12.16} $\in$ S.

min {x, y} = {0.2, 0.39, 1.23, 6.25,10.42} $\in$ S.

Thus T = {x, y, min {x, y}} is a subsemigroup of S.

We can define min on SS([0, 21)) and S is a super subset interval semigroup.

Thus if SS([0, n)) is the collection of all subsets of [0, n) then with min operation defined on SS([0, n)) we get a semigroup.

Likewise we can define max on SS([0, n)). We first claim all the four semigroups are distinct.

This claim will be illustrated by an example or two.

Example 2.90: Let $S_\cup$ = {SS([0, 12)), $\cup$} , $S_\cap$ = {SS([0, 12))A, $\cap$}, S_{min} = {SS([0, 12)), min} and S_{max} = {SS([0, 12)), max} be the four super interval subset semigroups.

Let x = {0.3, 1, 4, 10.8} and y = {0.5, 9, 4, 7.4} $\in S_\cup$, $S_\cap$, S_{min} and S_{max} .

Now for x, y $\in S_\cup$,

$x \cup y$ ={0.3, 1, 10.8, 7.4, 4, 9, 0.5} --- I

For x, y $\in S_\cap$, $x \cap y$ = {4} $\in S_\cap$ --- II

For x, y $\in S_{min}$,

min {x,y} = {0.3, 0.5, 1, 9, 4, 7.4} …. III

For x, y $\in S_{max}$,

max {x, y} = {0.5, 9, 4, 7.4, 10.8} …. IV

It is clearly evident all the sets I, II, III and IV are distinct. Thus all the four semigroups are different.

Thus we get four distinct semigroups on SS([0, n)) all of them are commutative.

In all these semigroups every element in SS([0, n)) is a subsemigroup of order one.

Now we can get yet another super subset interval semigroup using SS([0, n)) under usual ×.

Let x = {0.3, 7.2, 9.4, 8}
and y = {0.5, 8, 4, 0.01} ∈ SS([0, 10)).

x × y = {0.15, 3.6, 4.7, 4, 2.4, 57.6, 64, 75.2, 1.2, 28.8, 37.6, 32, 0.003, 0.072, 0.094, 0.08}

= {0.15, 3.6, 4.7, 4, 2.4, 7.6, 5.2, 1.2, 8.8, 2, 0.003, 0.072, 0.094, 0.08} ∈ SS([0, 10)).

Thus {SS([0, n)), ×} is a super subset interval semigroup of infinite order.

Here we see elements in general are not idempotents secondly we cannot complete subsets to get a subsemigroup.

We have only finitely many subsemigroups.

We have only finitely many subsemigroups of finite order.

Let x = {0.01, 0.1, 2} ∈ SS([0, 5));

x^2 = {4, 0.01, 0.0001, 0.001, 0.2, 0.02} ∈ SS([0, 5))

x^3 = {3, 0.02, 0.0002, 0.002, 0.04, 0.04, 0.4, 0.001, 0.00001, 0.0001, 0.000001} and so on.

We see even x generates an infinite order super subset interval subsemigroup.

DEFINITION 2.5: *P = {SS([0, n)), ×} is defined to be a super subset interval semigroup of infinite order.*

We will illustrate this situation by some examples.

Example 2.91: Let M = {SS([0, 10)), ×} be the super subset interval semigroup.

$x = \{5.1, 3.8, 6.007, 8.15\}$ and $y = \{5, 0.4, 0.9, 0.8, 2\} \in M$.

$x \times y = \{5.5, 9, 0.035, 0.75, 0.04, 1.42, 2.4028, 3.260, 4.59, 3.42, 5.4063, 7.335, 4.08, 3.04, 4.8056, 4.920, 0.2, 7.2, 2.014, 6.30\} \in M$.

This is the way product is performed. We see M has zero divisors. M also has idempotents.

For $x = \{0, 1, 5, 6\} \in M$ is such that $x \times x = x$.

Example 2.92: Let $B = \{SS([0, 12)), \times\}$ be the super subset interval semigroup under $\times$.

For $x = \{3, 6, 9\}$ and $y = \{4, 8, 0\} \in B$ we see

$x \times y = \{0\}$ so we have zero divisors in B.

Suppose $x = \{3.5, 2.8, 7.2, 8.1\}$ and $y = \{5, 10, 6\} \in B$

$x \times y = \{5.5, 2.0, 0, 4.5, 3, 4, 9, 4.8, 1.2, 0.6\}$.

Let $x = \{0.01\} \in B$ we see $x^2 = \{0.0001\}$, $x^3 = \{0.000001\}$ and so on.

Thus singleton set is also not a subsemigroup.

Let $V = \{\text{All subsets from } Z_{12}\} \subseteq B$ is a subsemigroup.

Let $x = \{0.5, 2, 0.4, 10\} \in B$.

We see x does not generate a subsemigroup of finite order.

For $x^2 = \{0.25, 1, 0.2, 5, 4, 0.8, 8, 0.16\}$,

$x^3 = \{0.125, 0.5, 1, 2.5, 8, 0.4, 4, 0.08, 2, 0.16, 0.32, 1.6, 3.2, 0.064, 10\}$ and so on.

We see $x \not\subseteq x^2$; $x \not\subseteq x^3$ and so on. We find the cardinality of x^n increases with the increasing n.

Example 2.93: Let M = {SS([0, 7)), ×} be the super interval subset semigroup. SS([0, 7)) has zero divisors and units.

We see if x = {0.1, 0, 0.8, 0.4} and y = {1.31, 2.52, 7.09} ∈ M then

x × y = {0.131, 0.252, 0.709, 0, 1.048, 2.016, 5.672, 0.524, 1.408, 2.836} ∈ M.

This is the way × operation is performed on M.

Now we proceed onto define super subset interval semiring using SS([0, n)).

We know {SS([0, n)), min} is a super subset interval semigroup.

Similarly {SS([0, n)), max} is a super interval subset semigroup.

Hence {SS([0, n)), min, max} is obiviously super interval subset semiring of infinite order.

We will illustrate this situation by some examples.

Example 2.94: Let S = {SS([0, 15)), min, max} be super subset interval semiring.

Let x = {0.7, 4.883, 9.12, 10.7, 12.4} and
y = {0.9, 0.212, 4.73, 14.32} ∈ S.

min {x, y} = {0.7, 0.212, 0.9, 4.73, 9.12, 10.7, 12.4}

and max {x, y} = {0.9, 4.73, 14.32, 4.883, 9.12, 10.7, 12.4} ∈ S. This is the way min, max operations are performed on S.

Clearly every pair $\{\{0\}, x\} \subseteq S$ is a subsemiring as $\min\{x, x\} = x$, $\min\{x, 0\} = 0$ and $\max\{x, x\} = x$ and $\max\{x, 0\} = x$.

Thus $\{\{0\}, x = \{0.92, 10.4, 14.5, 3.702, 9.37\}\} = P \subseteq S$ is a subsemiring of order two. However pair of elements in general are not subsemirings.

For if $x = \{14.371, 10.3, 4\}$, $y = \{2.1, 5.7, 11.2\}$ and $z = \{0\} \in S$.

We see $M = \{x, y, z = \{0\}\}$ is not a susemiring as $\min\{x, y\}$ and $\max\{x, y\} \notin M$.

Consider $\min\{x, y\} = \{2.1, 5.7, 11.2, 10.3, 4\} \notin M$ and

$\max\{x, y\} = \{14.371, 11.2, 5.7, 10.3, 4\} \notin M$.

Thus $M_C = \{x, y, z, \min\{x, y\}, \max\{x, y\}\} \subseteq S$ is the subsemiring defined as the completion of the set M.

Every subset of S can be completed to get a subsemiring. We have several subsemirings of any desired order.

Let $x = \{7.3, 4.5, 1.2\}$, $z = \{0\}$, $y = \{2.7, 5.8, 1\}$ and $u = \{4.7, 3.8, 9\} \in S$. $T = \{x, y, z, u\}$.

$\min\{x, y\} = \{2.7, 5.8, 1, 4.5, 1.2\} \notin T$,

$\min\{x, u\} = \{4.7, 3.8, 7.3, 4.5, 1.2\} \notin T$,

$\max\{x, y\} = \{7.3, 5.8, 1.2, 2.7, 5.8\} \notin T$,

$\max\{x, u\} = \{7.3, 9, 4.5, 4.7, 3.8\} \notin T$,

$\min\{y, u\} = \{2.7, 4.7, 3.8, 1, 5.8\} \notin T$ and

$\max\{y, u\} = \{4.7, 3.8, 9, 5.8\} \notin T$.

Thus T_C = {x, y, z, u, min {x, y}, min {x, u}, min {y, u}, max{x, y}, max {x, u}, max {y, u}} $\subseteq$ S is the completed subsemiring of the subset T.

Example 2.95: Let S = {SS([0, 21)) min, max} be the super subset interval semiring. S has subsemirings of order two, three and so on.

Let {x = {0.7, 3.9}
y = {12.9, 14.7} and z = {0}} = P.

min {x, y} = {0.7, 3.9} = x and
max {x, y} = {12.9,14.7} = y.

P is a subsemiring of order 3.

Example 2.96: Let S = {SS([0, 17)), min, max} be the super subset interval semiring. o(S) is infinite. S has subsemirings of all orders. {{0}, x} = P for every x $\in$ S is a subsemiring of order two.

We say two elements x, y $\in$ S are specially min or (max) orderable if min {x, y} = x and max {x,y} = y

and x $\leq_{min}$ y and y $\geq_{max}$ x.

For example if x = {10.3, 4.7, 8} and y = {12.5, 15.9, 16.99} $\in$ S.

min {x, y} = x and max {x, y} = y.

So this pair x, y is specially both min and max orderable. We cannot say any pair of elements x, y $\in$ S is specially min or max orderable.

Let x = {3.8, 10.3, 11.2} and y = {2, 15.7,12.5},

we see min {x, y} = {2, 3.8, 10.3, 11.2} and
max {x, y} = {3.8, 15.7, 12.5, 10.3, 11.2}.

So x and y are not specially min (and max) orderable. Thus we can say S is partially orderable with respect to min and max operation.

Example 2.97: Let S = {SS([0, 12)), min, max} be the super subset interval semiring. S has subsemirings of both finite and infinite order.

THEOREM 2.1: *Let M ={SS([0, n)), min, max} be the super subset interval semiring.*

(1) M has subsemirings of both finite and infinite order.
(2) M is a specially partially ordered set.
(3) No finite subsemiring is an ideal.

Proof is left as an exercise to the reader.

Next we proceed onto define super subset interval semiring under $\cup$ and $\cap$.

DEFINITION 2.6: *S = {SS([0, n)), min, max} is defined as a the super subset interval semiring.*

This is illustrated by examples.

Example 2.98: Let S = {SS([0, 7)), $\cup$, $\cap$} be the super subset interval semiring.

Let x = {0.3, 4.2,2.5, 4.801, 0.001} and

y ={4.801, 0.3, 2.5, 0.9918,1.082, 2.01,3.42} $\in$ S.

x $\cap$ y = {4.801, 0.3, 2.5} and x $\cup$ y = {0.3, 4.2,2.5, 4.801, 0.001, 0.9918, 1.082, 2.01, 3.42} $\in$ S.

Further min {x, y} = {0.3, 4.2, 2.5, 1.082, 2.01, 3.42, 0.9918, 4.801} and

max {x,y} ={4.801, 0.3, 2.5, 0.9918, 1.082, 2.01,3.42, 4.2}.

Thus {SS([0, 7)), $\cap$, $\cup$} is a different semiring from {SS([0, 7)), max, min}.

Thus study of {SS([0, 7)), $\cup$, $\cap$} happens to be an interesting problem as it is a Boolean algebra of infinite order as SS([0, 7)) is of infinite order.

Every singleton element with subset {0} is a subsemigroup. For if P = {x, {0}}, x $\cap$ {0} = 0 and x $\cap$ x = x $\cup$ x = x, x $\cup$ {0} = x.

Hene the claim.

This semiring also has infinite number of subsemirings of finite order none of them are ideals.

Example 2.99: Let S = {SS([0, 21)), $\cup$, $\cap$} be the super subset interval semiring of infinite order.

Now we can define the notion of super subset interval pseudo semiring.

S = {SS([0,n)), min, $\times$} is defined as pseudo super subset interval semiring. We call the semiring as pseudo since a $\times$ min {c, d} $\neq$ min {a $\times$ c, a $\times$ d} for a, c, d $\in$ SS([0, n)).

We will illustrate this situation by some examples.

Example 2.100: Let S = {SS([0, 20)), min, $\times$} be the super interval subset pseudo semiring.

Let x ={0.37, 4.8, 5.7, 10.8, 14.9} and
y = {5, 0, 10, 9,6} $\in$ S.

min {x, y} = {0.37, 0, 4.8, 5, 5.7, 10, 1.9, 6} and

x $\times$ y = {0, 18.5, 4.0, 8.5, 14, 14.5, 3.7, 8, 17,8, 9, 3.33, 3.2,11.3, 17.2, 14.1, 2.22, 8.8,14.2,4.8, 9.4}.

Now we show in general
$a \times \min \{c, d\} \neq \min \{a \times c, a \times d\}$.

Let a = {6.31, 7.52, 14.3}, c = {15.5, 18.7,10.8, 16.5} and d = {19.2,16.8} $\in$ S.

Consider $a \times \min \{c, d\}$

= a × {15.5, 18.7, 10.8, 16.5}

={6.31, 7.52, 14.3} × {15.5, 18.7, 16.8, 10.8, 16.5}

= {17.805, 17.997, 6.008, 8.148, 4.115, 16.56, 0.624, 6.336, 1.216, 4.08, 1.65, 7.41, 0.24, 14.44, 15.95} … I

min {a×c, a×d} = min {{6.31, 7.52, 14.3} × {5.5, 18.7,10.8, 16.5}, {6.31, 7.52, 14.3} × {19.2, 16.8}}

= min {{17.805, 17.997, 8.148, 4.115, 16.56, 0.624, 1.216, 4.08, 1.65, 7.41, 14.44, 15.95}, {1.152, 6.008, 4.384, 6.336, 14.56. 0.24}}

= {1.152, 6.008, 4.384, 6.336, 14.56, 0.24, 8.148, 4.115, 0.624, 1.216, 4.08, 1.65,7.41, 14.44} … II

Clearly I and II are distinct hence
$a \times \min \{c, d\} \neq \min \{a \times c, a \times d\}$.

Thus the semiring is only a pseudo semiring.

This pseudo semiring has zero divisors and idempotents.

Let x = {0,5} $\in$ S; x × x = x and

y = {0, 16} $\in$ S is such that y × y = y are idempotents.

Example 2.101: Let S = {SS([0, 11)), ×, min} be the super subset interval subset pseudo semiring.

Let x = {10} ∈ S

we see x^2 ={1} = {1} is the unit.

For if x = {1} we see y × {1} = y for every y ∈ S.

Let x = {4} then y = {3} ∈ S is such that x × y = {1}.

x = {6} and y = {2} ∈ S is such that x × y = {1}.

That is x = {6} is the inverse of y = {2} and vice versa.

x ={1, 11} is such that x^2 = x thus x is an idempotent.

y = {1, 6, 2}∈ S is such that y^2 = {1, 6, 2, 4, 3} ≠ y.

Let x = {9} and y = {5} ∈ S is such that xy = {1}.

Example 2.102: Let S = {SS ([0, 24)), min, ×} be the super interval subset pseudo semiring.

S has finite subsemirings which are not pseudo.

Let M_1 = {{0},{12}, {0, 12}}⊆ S be a super interval subset semiring of order 3 which is not pseudo.

M_2 = {{0}, {12}, {1}, {12, 1}, {12, 0}, {1, 0}, {0, 1, 12}} is again a super interval subset semiring of finite order which is not pseudo.

M_3 = {{0}, {6}, {12}, {18}, {0, 6}, {0, 12}, {0, 18}, {12, 6}, {18, 6}, {12, 18}, {0, 6, 12}, {0,6, 18}, {0, 6, 12, 18}, {6, 12, 18}, {0, 12, 18}} ⊆ S is a subsemiring of finite order.

We can get several such finite subsemirings of S all of them are not pseudo.

Now we proceed onto define super subset interval pseudo rings.

DEFINITION 2.7: *$S = \{SS([0, n)), +, \times\}$ be defined as the super subset interval pseudo ring if;*

(i) $\{SS([0, n)), +\}$ is an additive abelian super interval subset group.
(ii) $\{SS([0, n)), \times\}$ is a super subset interval semigroup which is commutative.
(iii) $a \times (b + c) \neq a \times b + a \times c$ in general for $a, b, c \in S$.

We will illustrate this situation by some examples.

Example 2.103: Let S = {SS([0, 10)), +, ×} be the super subset interval pseudo ring.

Let a = {0.31, 0.57, 9.2} and b = {0, 8.1, 4.5} ∈ S.

a + b = {0.31, 9.2, 0.57, 8.42, 8.67, 7.3, 4.81, 5.07,3.7} ∈ S.

a × b ={0.31, 0.57, 9.2} × {0, 8.1, 4.5}

= {0, 2.511, 1.395, 4.617, 2.565, 4.52,1.4} ∈ S.

Let c = {2.8, 5.7, 7} ∈ S;

a × (b + c) = a × ({0, 8.1, 4.5} +{2.8, 5.7, 7})

= a × {2.8, 5.7, 7, 0.9, 7.3, 3.8, 0.2,5.1, 1.5}

= {0.31, 0.57, 9.2} × {2.8, 5.7, 7, 0.9, 7.3, 3.8, 0.2, 5.1, 1.5}

= {0.868, 1.767, 2.17, 0.279, 2.263, 1.178, 0.062, 1.581, 0.465, 1.596, 3.249, 3.99, 0.513, 4.161, 2.166, 0.114, 2.907, 0.855, 5.76, 2.44, 4.4, 8.28, 7.16, 4.96, 1.84, 6.92, 3.8} … I

Consider a × b + a × c

= {0.31, 0.57, 9.2} × {0, 8.1, 4.5} + {0.31, 0.57,9.2} × {2.8, 5.7,7}

= {0, 2.511, 1.395, 4.617, 2.565, 4.52, 1.4} + {0.868, 1.767, 2.17, 1.596, 3.249, 3.99, 5.76, 2.44, 4.4}

={0.868, 1.767, 2.17, 1.596, 3.249, 3.99, 5.76, 2.44, 4.4, 3.379, 4.278, 4.681, 4.107, 5.760,6.501, 8.271, 4.951, 6.911, and so on} --- II

It is left as an exercise for the reader to find the rest of the elements.

However I and II are distinct so a × (b + c) ≠ a × b+ a × c for this a, b, c in S.

Thus the ring is pseudo.

Example 2.104: Let S = {SS([0, 19)), +, ×} be the super subset interval pseudo ring. S is S-super subset interval pseudo ring.

For P {{0}, {1}, {2}, …, {18}} ⊆ S is a field.

Hence the claim.

S has subrings which are not pseudo of finite order.

Example 2.105: Let S = {SS([0,7)), +, ×} be the super subset interval pseudo ring. S is a Smarandache super subset interval pseudo ring.

Example 2.106: Let S = {SS([0,24)), +, ×} be the Smarandache super subset interval pseudo ring.

Now using these super subset interval semigroups, groups, semirings, pseudo semirings and pseudo rings we build matrices and introduce algebraic structures on them.

These will be illustrated by some examples.

Example 2.107: Let
B = $\{(a_1, a_2, \ldots, a_7) \mid a_i \in \{SS([0, 12)), +\}, 1 \leq i \leq 9, +\}$ be the super subset interval row matrix group. B has infinite order subgroups. B also has finite order subgroups and B is commutative.

Example 2.108: Let

$$T = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{13} & a_{14} & a_{15} \end{bmatrix} \middle| \; a_i \in \{SS([0, 19)), +\}, 1 \leq i \leq 15, + \right\}$$

be the super subset interval subset matrix group.

Example 2.109: Let

$$M = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} \middle| \; a_i \in \{SS([0, 5)), +\}, 1 \leq i \leq 5, + \right\}$$

be the super subset interval matrix group.

We show how operations are performed on M.

$$\text{Let } A = \begin{bmatrix} \{0, 3.2, 1.5, 1.02\} \\ \{3, 0.52, 1.7\} \\ \{2, 0\} \\ \{3.12, 0.571, 4\} \\ \{1.2, 3.003, 4.2\} \end{bmatrix} \text{ and}$$

$$B = \begin{bmatrix} \{4.2, 3.5\} \\ \{0.2, 1\} \\ \{0, 0.6\} \\ \{0.2\} \\ \{1.2, 3.8\} \end{bmatrix} \in M.$$

$$A + B = \begin{bmatrix} \{4.2, 3.5, 2.4, 1.7, 0.7, 0, 0.22, 4.52\} \\ \{3.2, 4, 0.72, 1.52, 1.9, 27\} \\ \{0, 2, 0.6, 2.6\} \\ \{3.22, 0.77, 4.2\} \\ \{2.4, 4.203, 0.4, 0, 1.803, 3\} \end{bmatrix}.$$

This is the way operations are performed on M.

Example 2.110: Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \middle| \; a_i \in \{SS([0, 6)), +\}, 1 \leq i \leq 4, + \right\}$$

be the super subset interval matrix group.

$$\text{Let } x = \begin{bmatrix} \{0.3, 5.2, 0.6, 1.7\} & \{2.4, 0.8, 4.3, 0.7\} \\ \{0, 4.3, 5.8\} & \{1, 2, 4.21\} \end{bmatrix} \text{ and}$$

$$y = \begin{bmatrix} \{0.8, 3.71.4.52, 4\} & \{0, 0.3, 3.4, 4.32, 2.1\} \\ \{1, 1.23, 0.42, 0.5\} & \{0, 0.6, 1, 3\} \end{bmatrix} \in P.$$

It is easily verified x + y∈ P.

For every x ∈ P we have a unique y ∈ P such that

$$x + y = \begin{bmatrix} \{0\} & \{0\} \\ \{0\} & \{0\} \end{bmatrix}.$$

Example 2.111: Let S ={(a_1 | a_2 a_3) | a_i ∈ {SS([0, 10)), +}, 1 ≤ i ≤ 3, +} be the super subset interval matrix group of super row matrices.

Let A = ({0.9, 9.2, 1.6, 6.4, 2.7} | {5.3, 4.2} {7.8, 1.2, 3}) and B = ({0.9, 5.2,6.4} | {7.6, 8.7}, {9.221, 6.45}) ∈ S,

–A = ({9.1, 0.8, 8.4, 3.6, 7.3} | {4.7, 5.8} {2.2, 8.8,7}) ∈ S is such that A + –A = ({0}, {0}, {0}).

A + B = {(1.8, 6.1, 7.3, 0.1, 4.4, 5.6, 2.5, 6.8, 8, 7.3, 1.6, 2.8, 3.6, 7.9, 9.1} |{2.9, 4, 1.8, 2.9} {7.021,4.25,0.421, 7.65, 2.221, 9.45}) ∈ S.

This is the way "+" operation is performed on S.

Example 2.112: Let

$$M = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \middle| \; a_i \in SS([0, 5)), 1 \le i \le 3, + \right\}$$

be the super subset interval matrix group.

$$\text{Let } x = \begin{bmatrix} \{0.8, 4.7, 4.001, 3.2, 4, 2, 2.7, 1\} \\ \{0.3, 3, 3.33\} \\ \{2, 2.22, 4.2, 2.8\} \end{bmatrix} \text{ and}$$

$$y = \begin{bmatrix} (3,2,1) \\ (2.111, 4, 2) \\ (4, 4.44, 4.82) \end{bmatrix} \in M;$$

$$x + y = \begin{bmatrix} \{3.8, 2.7, 2.001, 1.2, 2, 0, 0.7, 4, \\ 2.8, 1.7, 1.001, 0.2, 1, 4.7, 3, 1.8, \\ 0.7, 0.001, 4.2, 3.7.2\} \\ \{2.411, 0.111, 0.441, 4.3, 2, \\ 2.33, 2.3, 0, 0.33\} \\ \{1, 1.22, 3.2, 1.8, 1.64, 3.64, \\ 2.24, 1.82, 4.2, 3.62, 2.04\} \end{bmatrix}$$

is in M. This is the way '+' operation is performed on M. This group has subgroups of both finite and infinite order.

Next we proceed onto give examples of all the five types of matrix super subset interval semigroups.

Example 2.113: Let M = $\{(a_1, a_2, a_3, a_4) \mid a_i \in \{SS([0, 7))), \min\}, 1 \leq i \leq 4, \min\}$ be the subset interval matrix semigroup. S is of infinite order.

S has subsemigroups.

If x = ({0.7, 4, 2}, {0.221, 5} {0, 0.714, 0.001}, {0.8}) and y = ({0.4, 5}, {0, 0.1} {0.8, 1, 4.32} {0.5, 0.2, 4}) ∈ S.

min {x, y} = ({0.4, 0.7, 4, 2}, {0, 0.1}, {0, 0.001, 0.714}, {0.2, 0.5, 0.8} ∈ M.

This is the way min operation is performed.

For all x ∈ M, min {x, x} = x for all x ∈ M and (0) = ({0}, {0}, {0}, {0}) ∈ M.

min {x, (0)} = ({0}¸{0}, {0}, {0}) = (0).

Example 2.114: Let

$$N = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} \middle| \; a_i \in SS([0, 12)), 1 \leq i \leq 6, \min \right\}$$

be the super interval subset semigroup of column matrices under the min operation.

$$\text{Let } x = \begin{bmatrix} \{0.4, 4.2\} \\ \{0\} \\ \{0.78, 7, 1\} \\ \{4.3, 5, 0\} \\ \{0.2, 0\} \\ \{8, 0.41, 001\} \end{bmatrix} \text{ and } y = \begin{bmatrix} \{0\} \\ \{1, 2.1\} \\ \{1.7, 0.004\} \\ \{0\} \\ \{1.2, 0.005\} \\ \{9, 10, 0.002\} \end{bmatrix} \in N.$$

$$\min \{x, y\} = \begin{bmatrix} \{0\} \\ \{0\} \\ \{0.78, 1, 0.004\} \\ \{0\} \\ \{0.2, 0.005, 0\} \\ \{8, 0.41, 0.001, 0.002\} \end{bmatrix} \in N.$$

This is the way min operation is performed on N.

Example 2.115: Let

$$B = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \middle| \; a_i \in S([0, 6)), 1 \leq i \leq 9, \min \right\}$$

be the super subset interval matrix semigroup under min operation. Let

$$x = \begin{bmatrix} \{8, 2.1, 0\} & \{0.5, 6.2\} & \{0\} \\ \{0.116, 0.001\} & \{5, 10\} & \{0.001, 0.2\} \\ \{0, 0.0045, 1.23\} & \{0\} & \{0.72, 0.117\} \end{bmatrix} \text{ and}$$

$$y = \begin{bmatrix} \{1.2, 0.116\} & \{0.201, 0.45\} & \{0.113, 7.8, 6\} \\ \{0.112, 0.1\} & \{0.7, 0.813, 0.91\} & \{0.0004\} \\ \{0.4, 0.52\} & \{0.74, 8, 9.331, 10\} & \{0.315, 0.113\} \end{bmatrix} \in B$$

min {x, y} =

$$\begin{bmatrix} \{1.2, 0.116, 0\} & \{0.201, 0.45\} & \{0\} \\ \{0.112, 0.1, 0.001\} & \{0.7, 0.813, 0.91\} & \{0.004\} \\ \{0, 0.0045, 0.4, 0.52\} & \{0\} & \{0.315, 0.113, .0.117\} \end{bmatrix}$$

This is the way min operation is performed on B.

Example 2.116: Let M = $\{(a_1, a_2, a_3, a_4, a_5) \mid a_i \in SS([0, 40)), 1 \leq i \leq 5, \max\}$ be the super subset interval semigroup of row matrices under max operation.

Let x = ({0}, {0.8, 0.9, 17, 8} {1, 0.001}, {14,19, 24,0.111} {0.1114, 2.8,14.5}) and

y = ({7.45, 9.3, 18.4, 27.4} {4, 0.3, 0.1125}, {0.,217, 4} {0.3, 0.1114, 0.00005}, {0.00000731, 14, 17.8, 14.6}) $\in$ M.

max {x, y} =({7.45, 9.3, 18.4, 27.4} {4, 0.8, 0.9, 17.8}, {4, 1, 0.217}, {14, 19, 24, 0.1114, 0.3}, {0.1114, 14, 17.8, 14.6, 14.5, 2.8}) $\in$ M.

This is the way max operation on the row interval subset matrix semigroup is performed.

Example 2.117: Let

$$N = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} \middle| \; a_i \in SS([0, 4)), 1 \leq i \leq 6, \max \right\}$$

be the super subset interval semigroup of column matrix under the max operation.

$$\text{Let } x = \begin{bmatrix} \{0\} \\ \{1, 1.4, 2.6, 1.7\} \\ \{3.21, 2.4, 3.1, 1.301\} \\ \{2.103, 1.001, 0.432\} \\ \{0.1107, 0.07\} \\ \{0\} \end{bmatrix} \text{ and}$$

$$y = \begin{bmatrix} \{3.21, 2.104, 1.116, 0.341, 2.0072\} \\ \{0\} \\ \{1, 2, 3, 1.24, 0.74\} \\ \{0.004, 0.0125, 3.4\} \\ \{4.9999\} \\ \{0.8, 4.3, 4.7, 3.201, 3, 2, 1\} \end{bmatrix} \in N.$$

$$\max\{x, y\} = \begin{bmatrix} \{3.21, 2.104, 1.116, 0.341, 2.0072\} \\ \{1, 1.4, 2.6, 1.7\} \\ \{3.21, 2.4, 3, 3.1, 2, 1.301\} \\ \{2.103, 3.4, 1.001, 0.432\} \\ \{4.9999\} \\ \{0.8, 4.3, 4.7, 3.201, 3, 2, 1\} \end{bmatrix}$$

is in N. This is the way max operation is performed.

Example 2.118: Let

$$T = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \right| a_i \in S([0, 20)), 1 \leq i \leq 4, \max\}$$

be the super interval subset matrix semigroup under max operation.

Let x = $\begin{bmatrix} \{0.8,18.4,19.8,0.2,4\} & \{19,16.8,4.02,0.001\} \\ \{16,,17.2,4.35,3.74,0\} & \{2,2.4,3,0.5,1\} \end{bmatrix}$ and

$$y = \begin{bmatrix} \{0.1,0.0003,19,0.00001\} & \{16,14.35,0.3333,4\} \\ \{16.8,2.1,4,5.9\} & \{16,14.3,19.99,18.32,7.5\} \end{bmatrix} \in T.$$

max {x, y} =

$$\begin{bmatrix} \{0.8,19,18.4,19.8,0.2,4\} & \{19,16.8,16,14.35,4.02,4,0.3333 \\ \{16.8,16,17.2,3.74,2.1,4.35,5.9,4\} & \{16,14.3,19.99,18.32,7.5\} \end{bmatrix}$$

$\in$ T.

This is the way max operation is performed on T.

Example 2.119: Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ \vdots & \vdots & \vdots & \vdots \\ a_{37} & a_{38} & a_{39} & a_{40} \end{bmatrix} \middle| \; a_i \in SS([0, 14)),\ 1 \leq i \leq 40, \max \right\}$$

be the super interval subset matrix semigroup. M has several subsemigroups of both finite and infinite order none of which are ideals.

Example 2.120: Let

$$M = \{(a_1, a_2, a_3) \mid a_i \in SS([0,6)),\ 1 \leq i \leq 3, \cap\}$$

be the super subset interval semigroup under the operation $\cap$ of row matrices.

Let x = ({0.8, 2,4, 5}, {0.61, 1.2, 5.7}, {0.93, 1.52, 0}) and y = ({0.8, 1, 2, 3, 4.5}, {0.3, 0.841, 1.2, 5, 5.3}, {{0, 1, 2, 3, 4.31}) ∈ M.

$x \cap y$ = { {0.8}, {1.2}, {0}) ∈ M.

We see every x ∈ M is a subsemigroup.

Every set {x, 0} ⊆ M is also a subsemigroup.

Example 2.121: Let

$$N = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} \middle| \; a_i \in SS([0, 10)), \; 1 \leq i \leq 5, \cap \right\}$$

be the super subset interval column matrix semigroup.

Every element x in B is a subsemigroup. Every element x in B is an idempotent.

We can for any given subset S ⊆ B complete S to form a subsemigroup.

Example 2.122: Let

$$P = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \middle| \; a_i \in S([0, 14)), 1 \leq i \leq 4, \cap \right\}$$

be the super interval subset semigroup.

$$\text{Let } x = \begin{bmatrix} \{7, 0.8, 4, 5\} & \{0\} \\ \{9.61, 6.8, 10.1\} & \{4.3, 5, 7, 1, 3, 2\} \end{bmatrix}$$

and $y = \begin{bmatrix} \{1,2,3,5,7\} & \{9,0.224,5,0.7\} \\ \{10.1,4,0,8.1\} & \{5,7,1,2,8.3\} \end{bmatrix} \in P$.

We see $x \cap y = \begin{bmatrix} \{7.5\} & \{0\} \\ \{10.1\} & \{1,2,5,7\} \end{bmatrix} \in P$.

We see every element in P is a subsemigroup.

Infact ideals of P of infinite order exist.

Example 2.123: Let

$$S = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} \middle| \, a_i \in SS([0, 12)),\ 1 \leq i \leq 5, \cup \right\}$$

be the super subset interval semigroup of column matrices.

S is of infinite order.

Every singleton element is a subsemigroup.

$$\text{Let } x = \begin{bmatrix} \{0.4,8,2.71\} \\ \{5.2,7.93,0.112\} \\ \{4.78,9.31,11.2106\} \\ \{4.01,3,2\} \\ \{8.41,5.32\} \end{bmatrix} \text{ and}$$

$$y = \begin{bmatrix} \{7,8,1.5\} \\ \{6,3.2,1.6\} \\ \{9.2,1.402,5.31\} \\ \{0.413,3.12,4.6\} \\ \{7,2,1,10\} \end{bmatrix} \in S.$$

$$x \cup y = \begin{bmatrix} \{0.4,8,2.71,7.15\} \\ \{5.2,7.93,0.112,6,3.2,1.6\} \\ \{4.78,9.31,11.2106,9.2,1.402,5.31\} \\ \{4.01,3,2,0.413,3.12,4.6\} \\ \{8.41,5.32,7,2,1,10\} \end{bmatrix} \in S.$$

We see $x \cup x = x$ for all $x \in S$.

Thus every element in an idempotent and a subsemigroup of order 1.

Example 2.124: Let P ={(a_1, a_2, a_3, a_4) | $a_i \in$ SS([0,9)), $1 \leq i \leq 4$, $\cup$} be the super interval subset semigroup of infinite order.

Let x = ({4.9, 0.91, 2.113, 7.2, 6.05} {0.37, 4.1116, 2.003,1.118}, {4.08, 5.13, 1.9}, {0.3106, 4, 0}) and

y = ({5, 5.201, 3.4, 8.01, 7}, {1, 2, 0.0001, 7.005}, {4, 2, 8.001, 5.021}, {5, 6, 4.02}) $\in$ P.

$x \cup y$ = ({4.9, 0.91, 2.113, 7.2, 6.05, 5, 5.201, 3.4, 8.01, 7}, {0.37, 4.1116, 2.003, 1.118,1, 2, 0.0001, 7.005}, {4.08, 5.13, 1.9, 4, 3, 8.001, 5.021}, {0.3106, 4, 0, 5, 6, 4.02})$\in$ P

This is the way $\cup$ operation is performed on P.

Every $x \in P$ is such that $x \cup x = x$, hence every singleton is a subsemigroup.

Example 2.125: Let

$$B = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ \vdots & \vdots & \vdots & \vdots \\ a_{37} & a_{38} & a_{39} & a_{40} \end{bmatrix} \middle|\; a_i \in SS([0, 40)), 1 \leq i \leq 40, \cup \right\}$$

be the super subset interval semigroup of infinite order. B has several infinite subsemigroups of infinite order which are not ideals of B.

Next we give examples of super subset interval semigroups under $\times$ using matrices.

Example 2.126: Let
$\{(a_1, a_2, a_3, a_4) \mid a_i \in SS([0,12)), 1 \leq i \leq 4, \times\}$ be the super interval subset semigroup. B has zero divisors.

Let x = ({0.3, 4.2, 0.7, 0.5, 0.01}, {0, 2.1, 4, 6.2} {10, 2.5, 4.3}, {1, 7, 8.2}) and y = ({10, 5, 6, 8, 0.2}, {0.1, 0.5, 0.06, 0.9}, {5, 8, 1, 0, 0.02}, {0, 0.7, 10}) $\in$ B.

x $\times$ y =({3, 6, 7,5, 1.5, 9, 3.5,2.5, 0.05, 1.8, 1.2,4.2, 3.5, 0.06, 2.4, 9.6, 5.64, 0.08, 0.06,0.84, 0.14, 0.1, 0.002}, {0, 0.21, 0.4, 0.62 1.05, 2, 3.1, 0.126,0.24, 3.72, 1.89, 3.6, 5.58}, {0, 2,0.5, 9.5, 8, 10.4, 10, 2.5, 4.3, 0.2, 0.05,0.086}, {0, 0.7, 4.9,5.64, 10}) $\in$ B.

This is the way product is performed. B has infinite number of zero divisors. B has both finite and infinite order subsemigroups.

Let $S_1 = \{\{(a_1, a_2, a_3, a_4) \mid a_i \in$ {Collection of all subsets of $Z_{12}\}, 1 \leq i \leq 4, \times\}$ be a subsemigroup of finite order.

$S_2 = \{\{(a_1, a_2, a_3, a_4) \mid a_i \in$ {Collection of all subsets from $\{0,6\}\}, 1 \leq i \leq 4, \times\}$ is again a subsemigroup of finite order.

$S_3 = \{\{(a_1, 0, 0, 0) \mid a_1 \in \{\text{Collection of all subsets from } \{0, 3, 6, 9\}\}, \times\}$ is again a subsemigroup of finite order.

Example 2.127: Let

$$P = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} \middle| \; a_i \in S([0, 6)), 1 \leq i \leq 6, \times_n \right\}$$

be the super subset interval column matrix semigroup.

P is of infinite order. P has infinite number of zero divisors and units.

Let

$$x = \begin{bmatrix} \{0.2, 0.8, 1.2, 5.6, 1.5, 2.14\} \\ \{0\} \\ \{5.3, 1.2, 1.75\} \\ \{0\} \\ \{0.2, 5, 4, 3, 0.4, 0.7, 0.8\} \end{bmatrix}$$

$$\text{and } y = \begin{bmatrix} \{5, 2, 3, 4\} \\ \{3.512, 3.2005, 1.205, 1.7071, 4.2013, 5.007\} \\ \{4, 5, 2, 1, 0.3, 0.5\} \\ \{4.321, 5.2004, 3.0099, 1.50091, 2.2216\} \\ \{4, 5, 0.2, 0.3\} \end{bmatrix} \in P.$$

$x \times_n y =$

$$\begin{bmatrix} \{1,4,0,2.0,1.5,4.70,0.4,1.6,2.4,5.2,3,4.28, \\ 0.6,2.4,3.6,4.8,4.5,0.42,0.8,3.2,4.8,4.4,2.56\} \\ \{0\} \\ \{3.2,4.8,1.00,0,2.5,2.75,4.6,2.4,3.50,5.3,1.2, \\ 1.75,1.59,0.36,0.515,2.65,2.75,0.875\} \\ \{0\} \\ \{0.8,2,4,0,1.6,2.8,3.2,1,2,3.5,4,0.04, \\ 0.8,0.6,0.08,0.14,0.16,0.06,1.5,1.2,0.90,0.12,0.21,0.24\} \end{bmatrix} \in P.$$

This is the way the product is performed.

Infact P has infinite number of zero divisors.

Example 2.128: Let

$$B = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \end{bmatrix} \middle| \; a_i \in S([0, 12)), 1 \le i \le 12, \times_n \right\}$$

be the super subset interval matrix semigroup.

$$\text{Let } x = \begin{bmatrix} \{0\} & \{6,2.11,3.5\} & \{1\} \\ \{0.11,0.26\} & \{1\} & \{5.21\} \\ \{1\} & \{3.2,4.5\} & \{0\} \\ \{0.45,1.2\} & \{0\} & \{7.2,5.6\} \end{bmatrix}$$

$$y = \begin{bmatrix} \{0.9, 9.312, 10.31, 7.63, 5.11\} & \{4, 6, 2, 5\} & \{0.37, 1.22, 1.35\} \\ \{5, 10, 7\} & \{4.5, 7.1, 8.57\} & \{1, 7\} \\ \{11.3, 10.75, 3.2, 5.7\} & \{2, 10, 5\} & \{0.7, 8.93, 4.37\} \\ \{0, 8, 10, 4, 5\} & \{9.2, 3.7, 4.89, 3.813\} & \{5, 10, 3\} \end{bmatrix} \in B$$

$x \times_n y =$

$$\begin{bmatrix} \{0\} & \{0, 8.44, 2, 0.66, 3, 4.22, 5.5\} & \{0.37, 1.22, 1.35\} \\ \{0.55, 1.30, 1.1, 2.6, 0.77, 1.82\} & \{4.5, 7.1, 8.57\} & \{5.21, 0.47\} \\ \{11.3, 10.75, 3.2, 5.7\} & \{7, 9, 8, 4, 10.5\} & \{0\} \\ \{0.45, 9.6, 3.6, 4.8, 1.4, 6, 3\} & \{0\} & \{0, 9.6, 8, 6, 4.8\} \end{bmatrix}$$

$\in$ B. This is the way product is performed on B. B has infinite number of zero divisors.

Next we give some examples of semirings of two types.

Example 2.129: Let S = $\{(a_1, a_2, a_3) \mid a_i \in SS([0, 10)), 1 \leq i \leq 3, \cup, \cap\}$ be the super interval subset matrix semiring.

Let x = ({5.31, 2.72, 1.003, 9.001}, {1, 2, 3, 8.003, 5.0251}, {8, 2, 4.371, 5.102}) and y = ({9.001, 2.72, 5, 8.3, 7.91}, {2, 3, 7, 9.00632, 1.3} {8, 4.371, 5, 3.331}) $\in$ S.

$x \cap y$ = ({9.001, 2.72}, {2, 3}, {8, 4.371}) $\in$ S.

This is the way '$\cap$' operation is performed on S.

$x \cup y$ = {(5.31, 2.72, 1.003, 9.001, 5, 8.3, 7.91} {1, 2, 3, 8.004, 5.0251, 7, 9.00632, 1.3}, {8, 2, 4.371, 5.102, 5, 3.331}) $\in$ S.

Thus S is a semiring.

Every element with ({0}, {0},{0}) is the subsemiring of order two.

Example 2.130: Let

$$M = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} \middle| a_i \in SS([0, 24)), 1 \le i \le 5, \cup, \cap \right\}$$

be the super subset interval semiring.

M has subsemirings of both finite and infinite order.

$$P_1 = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ 0 \\ 0 \\ 0 \end{bmatrix} \middle| a_1, a_2 \in SS([0, 24)), \cup, \cap \right\} \subseteq M$$

is a subsemiring as well as an ideal of M.

Infact M has atleast ${}_5C_1 + {}_5C_2 + \ldots + {}_5C_4$ distinct subsemirings which are ideals.

Example 2.131: Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \middle| \; a_i \in SS([0, 15)), \; 1 \leq i \leq 16, \cup, \cap \right\}$$

be the super interval subset semiring. M has ideals and subsemirings which are not ideals.

Example 2.132: Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & \ldots & a_{10} \\ a_{11} & a_{12} & \ldots & a_{20} \\ a_{21} & a_{22} & \ldots & a_{30} \\ a_{31} & a_{32} & \ldots & a_{40} \\ a_{41} & a_{42} & \ldots & a_{50} \end{bmatrix} \middle| \; a_i \in SS([0, 40)), \; 1 \leq i \leq 50, \cup, \cap \right\}$$

be the super interval subset matrix semiring.

We now give one or two examples of super subset interval matrix semiring under min and max operations.

Example 2.133: Let $M = \{(a_1, a_2, a_3, a_4, a_5) \mid a_i \in SS([0, 40)), 1 \leq i \leq 5, \min, \max\}$ be the super subset interval row matrix semiring.

Let x = ({10, 14.5, 8}, {0}, {14.7, 16, 18, 19}, {39, 2}, {0.5, 0.001}) and y = ({14, 6, 9}, {8, 9}, {12, 16}, {34.9, 12}, {0.0001, 0}) $\in$ M.

min {x, y} = ({10, 6, 9, 14, 8}, {0}, {12, 14.7, 16}, {2, 34.9, 12}, {0, 0.0001}) and

max {x, y} = ({14, 10, 14.5, 9}, {8, 9}, {14.7, 16, 18, 19}, {39, 34.9, 12}, {0.5, 0.001}) are in M.

This is the way operations are performed on M.

Every element x $\in$ M with
(0) = ({0}, {0},{0},{0},{0}) is a subsemiring of M.

Example 2.134: Let

$$M = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} \middle| \; a_i \in SS([0, 10)), 1 \leq i \leq 6, \min, \max \right\}$$

be a super subset interval column matrix semiring. M has ideals and subsemirings.

$$P_1 = \left\{ \begin{bmatrix} a_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \middle| \; a_1 \in \{\text{Collection of all subsets of } Z_{10}\}, \right.$$

$$\left. \min, \max \right\} \subseteq M$$

is a subsemiring of finite order. Clearly P_1 is not an ideal of M.

Example 2.135: Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ a_7 & \cdots & \cdots & \cdots & \cdots & a_{12} \\ a_{13} & \cdots & \cdots & \cdots & \cdots & a_{18} \\ a_{19} & \cdots & \cdots & \cdots & \cdots & a_{24} \\ a_{25} & \cdots & \cdots & \cdots & \cdots & a_{30} \\ a_{31} & \cdots & \cdots & \cdots & \cdots & a_{36} \\ a_{37} & \cdots & \cdots & \cdots & \cdots & a_{42} \\ a_{43} & \cdots & \cdots & \cdots & \cdots & a_{48} \end{bmatrix} \middle| \; a_i \in SS([0, 15)), \; 1 \le i \le 48, \min, \max \right\}$$

be the subset super interval semiring.

M has atleast $3({}_{48}C_1 + {}_{48}C_2 + \ldots + {}_{48}C_{48})$ number of finite super subset interval subsemirings.

Infact M has $({}_{48}C_1 + {}_{48}C_2 + \ldots + {}_{48}C_{47})$ number of infinite super subset interval subsemirings which are ideals.

Now we proceed onto describe pseudo super subset interval semirings.

Example 2.136: Let
$M = \{(a_1, a_2, a_3) \mid a_i \in SS([0, 4)), 1 \le i \le 3, \min, \times\}$ be the pseudo super subset interval semiring.

x = ({0.34, 2.41, 1.53}, {3.125, 1.631, 1.4211, 1, 0}, {1.5, 2.113, 2.431, 2.01}) and

y = ({0.6, 0.7, 2, 0.1, 0.4}, {3, 0, 0.9, 0.8, 0.2}, {1, 2, 3, 0.5, 0.6, 0.9}) $\in$ M.

min {x, y} = ({0.34, 0.1, 0.6, 0.7, 2, 0.1, 0.4}, {3, 0, 0.9, 0.8, 0.2, 1.631, 1.4211, 1, 0}, {1, 1.5, 0.5, 0.6, 0.9, 2, 2.431, 2.113, 2.01}) and

$x \times y$ = ({0.204, 1.446, 0.918, 0.238, 1.687, 1.071, 0.68, 0.82, 3.06, 0.034, 0.241, 0.153, 0.136, 0.964, 0.612}, {0, 1.375, 0.893, 0.2633, 3, 2.8125, 1, 1.27899, 1.4679, 2.250, 3.262, 2.8422, 2, 2.5, 1.3048, 1.13688, 0.8}, {1.5, 2.113, 2.431, 2.01, 3, 0.226, 0.862, 0.02, 0.5, 2.339, 3.293, 6.03, 0.75, 1.0565, 1.2155, 1.005, 0.9, 1.2678, 1.4586, 1.206, 1.35, 1.9017, 2.1879, 1.809}) $\in$ M.

This is the way the operations min and $\times$ are performed on M. It is easily verified that $a \times \min\{b, c\} \neq \min\{a \times b, a \times c\}$ in general for a, b, c $\in$ M.

Example 2.137: Let

$$B = \left\{\left.\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_9 \end{bmatrix} \right| a_i \in SS([0, 25)), 1 \leq i \leq 9, \min, \times_n\right\}$$

be the super subset interval column matrix pseudo semiring. B has subsemirings of finite order which are not pseudo subsemirings or ideals.

B also has pseudo subsemirings of infinite order which are ideals.

Example 2.138: Let

$$W = \left\{\left.\begin{bmatrix} a_1 & a_2 & \cdots & a_9 \\ a_{10} & a_{11} & \cdots & a_{18} \\ a_{19} & a_{20} & \cdots & a_{27} \\ a_{28} & a_{29} & \cdots & a_{36} \\ a_{37} & a_{38} & \cdots & a_{45} \\ a_{46} & a_{47} & \cdots & a_{54} \end{bmatrix} \right| a_i \in SS([0, 3)), 1 \leq i \leq 54, \min, \times_n\right\}$$

be the super subset interval pseudo semiring.

Now we proceed onto describe by examples the notion of super subset interval pseudo ring.

Example 2.139: Let W = $\{(a_1, a_2, a_3, a_4) \mid a_i \in SS([0, 15)), 1 \leq i \leq 4, +, \times\}$ be the super subset interval pseudo ring.

Let x = ({10.3, 7.5, 1.4}, {4.021, 3.152}, {13.215, 10.3151}, {8.131}), and

y = ({4, 6, 10, 2}, {0.8, 0.7}, {9, 10, 1}, {1, 2, 3, 5, 10}) $\in$ W.

x + y = {14.3, 11.5, 5.4, 1.3, 13.5, 7.4, 5.3, 2.5, 11.4, 12.3, 9.5, 3.4}, {4.821, 3.915, 4.721, 3.852}, {5.215, 4.3151, 8.215, 5.3151, 14.215, 11.3151}, {9.131, 10.131, 11.131, 13.131, 3.131}) $\in$ W.

x $\times$ y = ({11.2, 0, 5.6, 1.8, 8.4, 13, 14, 5.6, 2.8}, {3.2168, 2.5216, 2.8147, 2.2064} {11.8935, 3.7359, 12.15, 13.151, 13.215, 10.3151}, {9, 10, 1, 3, 5, 2, 12, 0}) $\in$ W.

This is the way operations are performed on W. W is a pseudo ring of infinite order.

Example 2.140: Let S =

$$\left\{ \left[\begin{array}{c|cc|ccc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ a_8 & \cdots & \cdots & \cdots & \cdots & \cdots & a_{14} \\ \hline a_{15} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{21} \\ a_{22} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{28} \\ a_{29} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{35} \\ \hline a_{36} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{42} \\ a_{43} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{49} \end{array} \right] \middle| \; a_i \in SS([0, 42)), \right.$$

$$1 \leq i \leq 49 +, \times \}$$

be a subset super interval pseudo ring.

Clearly S is non commutative and is of infinite order.

Example 2.141: Let

$$S = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \middle| \, a_i \in SS([0, 5)),\ 1 \leq i \leq 4, +, \times \right\}$$

be the super interval subset square matrix pseudo ring. S is non commutative.

$$\text{Let } x = \begin{bmatrix} \{2.1, 3.2\} & \{4.2, 2.5\} \\ \{0\} & \{1, 4\} \end{bmatrix}$$

$$\text{and } y = \begin{bmatrix} \{4, 2, 1\} & \{0.5, 0.6\} \\ \{0.7, 0.8\} & \{0.4, 0.1\} \end{bmatrix} \in S \text{ and}$$

$x \times y =$

$$\begin{bmatrix} \{1.34, 0.74, 3.14, 4.34, 1.14, & \{2.73, 1.47, 2.02, 3.28, \\ 0.04, 0.15, 4.55, 0.95, 1.56, & 2.94, 1.68, 3.60, 2.34, \\ 0.46, 3.15, 4.95, 3.85, & 2.05, 1.3, 2.6, 1.85, \\ 1.76, 1.16, 2.56, 4.76\} & 2.26, 2.92, 1.51, 2.27\} \\ \{0.7, 0.8, 2.8, 3.2\} & \{0.4, 1.6, 0.1\} \end{bmatrix} \in S.$$

Consider $y \times x =$

$$\begin{bmatrix} \{3.4, 2.8, 4.2, 1.4, 2.1, 3.2\} & \begin{matrix} \{0.5, 0.6, 2, 2.4, 2.3, \\ 3.8, 4.5, 4.2, 1.5, 1.6, \\ 4.9, 3, 3.4, 3.1, 3.9, 4, 0.4, \\ 0.8, 4.7, 4.8, 1.2, 1.6\} \end{matrix} \\ \{2.24, 1.47, 2.56, 1.68\} & \begin{matrix} \{0.7, 0.8\} \times \{4.2, 2.5\} + \\ \{0.4, 0.15\} \times \{4.1\} \end{matrix} \end{bmatrix}.$$

It is clear x × y ≠ y × x.

Thus S is a non commutative super subset interval matrix pseudo ring. S has zero divisors which are infinite in number.

However in the above example if we replace the product × by $\times_n$ the natural product, then x $\times_n$ y = y $\times_n$ x commutative and $\begin{bmatrix} \{1\} & \{1\} \\ \{1\} & \{1\} \end{bmatrix}$ is the unit or identity with respect to $\times_n$.

Example 2.142: Let

$$S = \left\{ \left[\begin{array}{ccc} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \hline a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \\ \hline a_{16} & a_{17} & a_{18} \\ \hline a_{19} & a_{20} & a_{21} \\ a_{22} & a_{23} & a_{24} \\ a_{25} & a_{26} & a_{27} \\ a_{28} & a_{29} & a_{30} \\ \hline a_{31} & a_{32} & a_{33} \end{array} \right] \middle| \; a_i \in SS([0, 14)), 1 \leq i \leq 13, +, \times \right\}$$

be the super interval subset column super matrix pseudo ring. B is commutative and of infinite order.

B has infinite number of zero divisors. B also contains super subset interval rings which are not pseudo.

Now we can construct super subset interval algebraic structures by replacing [0, n) in SS([0, n)) by C([0, n)) or ⟨[0, n) ∪ I⟩ or C(⟨ [0, n) ∪ I⟩).

This study is considered as a matter of routine so is left as an exercise to the reader. However we illustrate them by an example or two.

Example 2.143: Let $S = \{(a_1, a_2, a_3, a_4) \mid a_i \in SS([0, 7)\} =$ {Collection of all subsets of the subset C([0, 7))}, $1 \leq i \leq 4, +\}$ be the super subset interval group.

If on S, '$\cup$' operation is taken it is a semigroup. On similar lines under $\cap$ or $\times$ or min or max S is only a semigroup.

If two operations ($\cup$ or $\cap$) or {min and max} are taken S is a semiring. If min and $\cap$ is defined on S, S becomes a pseudo semiring.

Example 2.144: Let

$$M = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_9 \end{bmatrix} \middle| \, a_i \in SS(C\langle[0, 23) \cup I\rangle), 1 \leq i \leq 9, \min \right\}$$

be the super subset interval neutrosophic column matrix semigroup.

Example 2.145: Let

$$B = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{28} & a_{29} & a_{30} \end{bmatrix} \middle| \, a_i \in SS(C\langle[0, 8) \cup I\rangle), 1 \leq i \leq 30, \max \right\}$$

be the super subset interval finite complex modulo integer neutrosophic matrix semigroup.

Example 2.146: Let

$$V = \left\{\left.\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \end{bmatrix}\right| a_i \in SS(C[0, 4)), 1 \leq i \leq 12, \times_n\right\}$$

be the super interval subset semigroup of finite complex modulo integers.

V has subsemigroups, ideals and infinite number of zero divisors.

Example 2.147: Let

$$M = \left\{\left.\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{18} \end{bmatrix}\right| a_i \in SS(C[0, 14)), 1 \leq i \leq 18, \cup, \cap\right\}$$

be the super interval subset column matrix complex modulo integer semiring.

Example 2.148: Let

$$W = \left\{\left.\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \\ a_{11} & a_{12} \\ a_{13} & a_{14} \end{bmatrix}\right| a_i \in SS(C\langle[0, 5) \cup I\rangle), 1 \leq i \leq 14, \min, \max\right\}$$

be the super subset interval finite complex modulo integer neutrosophic semiring of matrices.

Example 2.149: Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \middle| \; a_i \in SS(C([0, 15)), 1 \leq i \leq 9, \min, \times \right\}$$

be the super subset interval pseudo semiring of finite complex modulo integers. Clearly M is non commutative.

Example 2.150: Let

$$N = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \middle| \; a_i \in SS(\langle [0, 20) \cup I \rangle), 1 \leq i \leq 16, \min, \times_n \right\}$$

be the super subset interval neutrosophic matrix pseudo semiring which is commutative.

Example 2.151: Let

$$V = \left\{ \begin{bmatrix} a_1 & a_2 & \dots & a_9 \\ a_{10} & a_{11} & \dots & a_{18} \\ a_{19} & a_{20} & \dots & a_{27} \\ a_{28} & a_{29} & \dots & a_{36} \end{bmatrix} \middle| \; a_i \in SS(\langle [0, 12) \cup I \rangle), 1 \leq i \leq 36, \min, \times_n \right\}$$

be the super interval subset complex finite modulo integer neutrosophic pseudo semiring which is commutative.

Example 2.152: Let

$$W = \left\{ \left. \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \right| a_i \in SS(\langle[0, 22) \cup I\rangle), \right.$$

$$1 \leq i \leq 33, \min, \times_n\}$$

be the super interval subset neutrosophic matrix pseudo semiring.

Example 2.153: Let

$$S = \left\{ \left. \left[\begin{array}{c} a_1 \\ \hline a_2 \\ a_3 \\ a_4 \\ \hline a_5 \\ a_6 \\ \hline a_7 \\ a_8 \\ a_9 \\ \hline a_{10} \\ \hline a_{11} \end{array} \right] \right| a_i \in SS(C\,[0, 13)), 1 \leq i \leq 11, \min, \times_n \right\}$$

be the subset super interval matrix finite complex modulo integer pseudo semiring.

Example 2.154: Let

$M = \{(a_1, a_2, a_3, a_4, a_5) \mid a_i \in SS(\langle[0, 23) \cup I\rangle), 1 \leq i \leq 5, +, \times\}$

be the subset interval neutrosophic row matrix pseudo ring. M has zero divisors.

Example 2.155: Let

$$S = \left\{ \left[\begin{array}{c} a_1 \\ a_2 \\ a_3 \\ \hline a_4 \\ \hline a_5 \\ a_6 \\ \hline a_7 \\ a_8 \\ a_9 \end{array} \right] \middle| \; a_i \in SS(C\langle[0, 10) \cup I\rangle), 1 \leq i \leq 9, +, \times_n \right\}$$

be the super subset interval finite complex modulo integer neutrosophic super column matrix pseudo ring. M has infinite number of zero divisors.

Example 2.156: Let

$$T = \left\{ \left[\begin{array}{cc|ccc} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & \cdots & \cdots & \cdots & a_{10} \\ \hline a_{11} & \cdots & \cdots & \cdots & a_{15} \\ a_{16} & \cdots & \cdots & \cdots & a_{20} \\ a_{21} & \cdots & \cdots & \cdots & a_{25} \\ \hline a_{26} & \cdots & \cdots & \cdots & a_{30} \end{array} \right] \middle| \; a_i \in SS(C[0, 19)), 1 \leq i \leq 30, +, \times_n \right\}$$

be the super subset interval finite complex number super matrix pseudo ring. T has infinite number of zero divisors.

Now we can built all the three types of vector spaces using SS([0, n)) or SS(C([0, n)), or SS(⟨[0, n) ∪ I⟩) or SS(C⟨[0, n) ∪ I⟩).

We will illustrate these situations by some examples.

Example 2.157: Let
$V = \{(a_1, a_2, \ldots, a_{10}) \mid a_i \in SS([0, 17)), 1 \leq i \leq 10, +\}$
be the super subset interval vector space over the field Z_{17}.

Example 2.158: Let

$$S = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{18} \end{bmatrix} \middle| \; a_i \in SS(C([0, 13)), 1 \leq i \leq 18, + \right\}$$

be the super subset interval finite complex modulo integer column matrix vector space over the field Z_{13}.

S can be a S-super subset interval finite complex vector space over $C(Z_{13})$.

S can be also realized as a pseudo S-vector space over the pseudo S-ring $\{[0, n), +, \times\}$.

S can be defined as super strong S-pseudo vector space over the S-super pseudo ring $\{SS([0, n)), +, \times\}$.

Example 2.159: Let

$$S = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \middle| \; a_i \in S((\[0, 11) \cup I\rangle), 1 \leq i \leq 9, +, \times \right\}$$

be the S-strong super subset interval pseudo linear algebra ove the S-super interval subset pseudo ring.

$R = \{SS(\langle[0, 11) \cup I\rangle), +, \times\}$.

Clearly on S we can define pseudo inner product and linear functionals.

If S is defined over Z_{11} or $\langle Z_{11} \cup I\rangle$ or $\{[0, 11), +, \times\}$ or $\{\langle[0, 11) \cup I\rangle, +, \times\}$ then both inner product and pseudo linear functional cannot be defined.

Example 2.160: Let

$$V = \left\{ \left. \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_9 \end{bmatrix} \right| a_i \in SS(C\langle[0, 13) \cup I\rangle), 1 \leq i \leq 9, +, \times_n \right\}$$

be the S-super subset interval finite complex modulo integer pseudo linear algebra over the S-super subset interval pseudo ring $R = \{SS(C\langle[0,13) \cup I\rangle); +, \times\}$.

On V we can define inner product.

Example 2.161: Let

$$M = \left\{ \left. \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \\ a_{17} & a_{18} & a_{19} & a_{20} \end{bmatrix} \right| a_i \in SS(C\langle[0, 5) \cup I\rangle), \right.$$

$$\left. 1 \leq i \leq 20, +, \times_n \right\}$$

be the S-super subset interval pseudo linear algebra over the S-super subset interval pseudo ring of finite complex neutrosophic modulo integers R={SS(C⟨[0, 5) ∪ I⟩), +, ×}.

M has infinite number of zero divisors.

Example 2.162: Let

$$V = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \middle| \; a_i \in SS(C\langle[0, 5) \cup I\rangle, +, \times), 1 \leq i \leq 3, +, \times_n \right\}$$

be the S-super subset interval pseudo linear algebra of complex finite modulo integers over the S-super subset interval pseudo ring of complex finite modulo integers R ={SS(C([0, 5)), +, ×}.

Define $\langle \;\; \rangle : V \times V \to R$; for

$$x = \begin{bmatrix} \{2i_F + 3, 0.3i_F, 2\} \\ \{0\} \\ \{0.32i_F + 0.43, 2\} \end{bmatrix}, \; y = \begin{bmatrix} \{2 + 3.1i_F, 0, 2i_F + 4\} \\ \{4, 3 + 2i_F\} \\ \{0.4 + 2i_F, 3i_F + 2\} \end{bmatrix} \in V.$$

$\langle x,y \rangle = \{2i_F + 3, 0.3i_F, 2\} \times \{2 + 3.1i_F, 0, 2 i_F + 4\} + \{0\} \times \{4, 3 + 2i_F\} + \{0.32i_F + 0.43, 2\} \times \{0.4 + 2i_F, 3i_F + 2\}$

$= \{4i_F + 6 + 6.2 \times 4 + 9.3i_F, 0, 0.93 \times 4 + 0.6i_F, 4 + 6.2i_F, 4 \times 5 + 6i_F + 12 + 8i_F, 0.6 \times 4 + 1.2i_F + 4i_F + 8\} + \{0\} + \{0.8 + 4i_F, 6i_F + 4, 0.128i_F + 1.72i_F + 0.64 \times 4 + 0.172 + 0.96 \times 4 + 1.29i_F + 0.64i_F + 0.86\}$

$= \{3.3i_F + 3.2, 0, 3.72 + 0.6i_F, 4 + 1.2i_F, 2 + 4i_F, 0.4 + 0.2i_F\} + \{0\} + \{0.8 + 4i_F, 4 + i_F, 1.848i_F + 2.722, 1.93i_F, 4.7\}$

$= \{0, 3.3i_F + 3.2, 3.72 + 0.6i_F, 4 + 1.2i_F, 2 + 4i_F, 0.4 + 0.2i_F, 0.8 + 4i_F, 4 + i_F, 1.848i_F + 2.722, 1.93i_F + 4.7, 4 + 2.3i_F, 2.2 + 4.3i_F, 0.148i_F + 1.4, 2.9 + 0.23i_F, 4.52 + 4.6i_F, 2.72 + 0.8i_F,$

$2.448i_F + 1.542$, $2.53i_F + 3.42$, $4.8 + 0.2i_F$, $3 + 2.2i_F$, $3.048i_F + 1.722$, $3.7 + 3.13i_F$, $2.8 + 3i_F$, 1, $0.848i_F + 4.722$, $1.7 + 0.93i_F$, $1.2 + 4.2i_F$, $4.4 + 1.21i_F$, $2.248i_F + 2.922$, $0.1 + 2.13i_F\} \in R$.

This is the way inner product is performed on V.

Example 2.163: Let

$$V = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \end{bmatrix} \middle| \; a_i \in SS(C\langle[0, 7) \cup I\rangle), 1 \leq i \leq 6, +, \times_n \right\}$$

be the S-super subset interval pseudo linear neutrosophic matrix pseudo linear algebra over the S-pseudo subset interval neutrosophic ring $R = \{SS\langle[0, 7) \cup I\rangle, +, \times\}$,

$\langle\, \rangle : V \times V \to R$ results is subsets in $SS\langle[0, 7) \cup I\rangle)$).

For if $x = \begin{bmatrix} x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 \end{bmatrix}$ and $y = \begin{bmatrix} y_1 & y_2 & y_3 \\ y_4 & y_5 & y_6 \end{bmatrix}$; $x, y \in V$ and $x_i, y_i \in R$.

$$\langle x, y\rangle = \sum_{i=1}^{6} x_i y_i = z \in R \text{ is a subset of } R.$$

Now we can define pseudo linear functional $f : V \to R$;

by $f(x) = x_1 + x_2 + x_3 = m \in R$ is a subset.

For instance if x =

$$\begin{pmatrix} \{0.7+2I, 6.1+5.2I \\ 0.17I+2, 4\} & \{0,1,I,2\} & \{3, 2I, 4+I \\ \{6+6I, 0.5I, 0.44I, \\ 0.67I+6.72\} & \{0\} & \{1.37I+2, \\ & & 0.721I+3\} \end{pmatrix}$$

f(x) = {0.7 + 2I, 6.1 + 5.2I, 0.17I + 2, 4} + {0, 1, I, 2}+ {3,2I, 4+I}

= {0.7 + 2I, 6.1 + 5.2I, 0.17I + 2, 4, 1.7 + 2I, 0.1 + 5.2I, 3 + 0.17I,5, 0.7 + 3I, 6.1+ 6.2I, 1.17I + 2, 4 + I, 2.7 + 2I,1.1+ 5.2I, 4 + 0.17I,6} +{3,2I,4 + I}

= 3.7 + 2I, 2.1 + 5.2I,5 + 0.17I,0, 4.7 + 2I, 3.1 + 5.2I, 6 + 0.17I, 1, 3.7 + 3I, 2.1 + 6.2I, 5 + 1.17I, I, 5.7 + 2I, 4.1 + 5.2I, 0.17I, 2, 0.7 + 4I, 6.1 +0.2I, 2.17I + 2, 2I + 4, 4I + 1.7, 0.2I +0.1, 2.17I + 3, 2I + 5, 5I +0.7, 1.2I + 6.1,2 + 3.17I,4 + 3I, 2.7 + 4I,4 + 3I, 0.2I+ 1.1, 4 + 2.17I, 6 + 2I, 4.7+ 3I, 3.1 + 6.2I,1.17I + 6, 1 + I, 5.7 + 3I, 4.1 + 6.2I, 1.17I, 2 + I, 4.7+ 4I, 3.1 + 0.2I, 2.17I + 6, 1 + 2I, 3I + 6.7, 6.2I + 5.1, 1 + 1.17I, 3 + I} $\in$ R.

This is the way linear functionals are defined. For linear functionals can be defined over the same ring from which entries are taken.

We can find subspaces, the concept of linear operators and linear transformations which is taken as a matter of routine for they can be carried out with appropriate modifications.

Example 2.164: Let

$$S = \left\{\left.\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}\right| a_i \in SS(C([0, 3)), 1 \le i \le 4, +\right\}$$

and

$$R = \left\{\left.\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \end{bmatrix}\right| a_i \in SS(C\langle[0, 13) \cup I\rangle), 1 \le i \le 6, +\right\}$$

be any vector spaces over Z_{13}.

We can define linear transformation from S $\to$ R as well as linear transformation from R to S.

Define T : S $\to$ R

$$\text{By } T(\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}) = \begin{bmatrix} a_1 & \{0\} & a_2 \\ a_3 & \{0\} & a_4 \end{bmatrix}.$$

T is a linear transformation from S to R.

We can also define linear transformation P from R to S.

$$\text{By } P\,(\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \end{bmatrix}) = \begin{bmatrix} a_1 + a_2 \\ a_3 \\ a_4 \\ a_5 + a_6 \end{bmatrix}$$

is a linear transformation from R to S.

Here a_i's are subsets from SS(C([0, 13)).

We can also define linear operators from S to S (R to R).

We just give an example of each.

T_1 : S $\to$ S defined

$$\text{by } T_1\,(\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}) = \begin{bmatrix} a_1 + a_2 \\ a_2 + a_3 \\ a_3 + a_4 \\ a_4 + a_1 \end{bmatrix} \in S.$$

T_1 is a linear transformation on S.

Define $P_1 : R \to R$ defined by

$$P_1 \left[\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \end{bmatrix}\right] = \begin{bmatrix} a_1 & a_2 & \{0\} \\ \{0\} & a_3 & a_4 \end{bmatrix}.$$

P_1 is a linear transformation on R. The study of the algebraic structure enjoyed by Hom (R,R) and Hom (R, S), Hom (S, R), Hom(S,S) is a matter of routine but is an interesting problem.

Example 2.165: Let
$B = \{(a_1, a_2, a_3, a_4) \mid a_i \in \{SS(C\langle[0, 23) \cup I\rangle), +, \times\}, 1 \leq i \leq 4\}$ be the S-super subset interval row matrix S-vector space over the S-neutrosophic pseudo ring, $R = \{\langle[0, 23) \cup I\rangle), +, \times\}$.

We can define $T : B \to B$ the collection of linear operators on B.

Example 2.166: Let

$$M = \left\{\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \end{bmatrix} \middle| \; a_i \in SS(C\langle[0, 19) \cup I\rangle), 1 \leq i \leq 10, +, \times_n\right\}$$

be the S-super pseudo subset interval neutrosophic matrix vector space (S-linear algebra) over the S-super subset interval pseudo ring $R = \{SS(\langle 0, 19) \cup I\rangle), +, \times\}$.

We can define linear functionals using M is an inner product space.

We suggest some problems for the reader, some at research level, some easy and some of them difficult.

Problems:

1. What are the special features enjoyed by the natural classes of interval algebraic structures built using S([0, n))?

2. Compare the algebraic structure on [0, n) with that of S([0, n)).

3. Prove ([0, n), max) $\subseteq$ [S([0, n)), max}.

4. Is problem 3 is true if max is replaced by min or ×?

5. Prove intervals of the form [a, b] and [b, a] must be taken for × to be compatible.

6. Let P = {S([0, 24)), max} be the interval subset semigroup.

   (i) Find all ideals which are not S-ideals.
   (ii) Can P have ideals which are not S-ideals?
   (iii) Can P have ideals of finite order?
   (iv) Prove P can have interval subset subsemigroups of finite order.
   (v) Prove P can have subset subsemigroup of infinite order.
   (vi) Can P have subsemigroup which is not a S-subsemigroup?

7. Let S = {S([0, 23)), max} be the interval subset semigroup.

   Study questions (i) to (vi) of problem 6 for this S.

8. Let M = {S([0, 48)), min} be the interval subset semigroup.

   Study questions (i) to (vi) of problem 6 for this M.

9. Let S = {S([0, 29)), min} be the interval subset semigroup.

   Study questions (i) to (vi) of problem 6 for this S.

10. Let T = {S([0, 12)), $\times$} be the interval subset semigroup.

    Study questions (i) to (vi) of problem 6 for this T.

11. Let B = {S([0, 19)), $\times$} be the interval subset semigroup.

    Study questions (i) to (vi) of problem 6 for this B.

12. Let L = $\{(a_1, a_2, \ldots, a_{10}) \mid a_i \in S([0, 10)), 1 \leq i \leq 10,$ min} be the interval subset semigroup.

    (i) Study questions (i) to (vi) of problem 6 for this L.
    (ii) Prove L has infinite number of zero divisors.
    (iii) Prove every element in L is an idempotent.
    (iv) Is the number of ideals in L finite or infinite in number?

13. Let S = $\{(a_1 \mid a_2\ a_3 \mid a_4\ a_5\ a_6 \mid a_7) \mid a_i \in S([0, 23)),$ $1 \leq i \leq 7$, min} be the interval subset semigroup of super row matrix.

    Study questions (i) to (vi) of problem 6 for this S.

14. Let W = $\left\{\begin{bmatrix} a_1 & a_2 & \dots & a_{20} \\ a_{21} & a_{22} & \dots & a_{40} \\ a_{41} & a_{42} & \dots & a_{60} \\ \vdots & \vdots & & \vdots \\ a_{61} & a_{62} & \dots & a_{80} \end{bmatrix} \middle| a_i \in S(C\langle[0, 43) \cup I\rangle);\right.$ $1 \leq i \leq 80$, min} be the interval subset matrix semigroup.

    Study questions (i) to (iv) of problem 12 for this W.

15. Let S = $\left\{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \\ a_9 \end{bmatrix} \middle| a_i \in S([0, 12)); 1 \leq i \leq 9, \min\right\}$ be the subset interval column matrix.

    Study questions (i) to (iv) of problem 12 for this S.

16. Let P = {$(a_1, a_2, \dots, a_{12})$ | $a_i \in S([0, 15))$; $1 \leq i \leq 12$, max} be the subset interval row matrix semigroup.

    Study questions (i) to (iv) of problem 12 for this P.

17. Let M = $\{(a_1\ a_2 \mid a_3\ a_4 \mid a_5\ a_6\ a_7 \mid a_8) \mid a_i \in S([0, 47));$ $1 \leq i \leq 8$, max$\}$ be the subset interval row super matrix semigroup.

    Study questions (i) to (iv) of problem 12 for this M.

18. Let L = $\left\{\left[\begin{array}{cccccc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ a_7 & a_8 & \dots & \dots & \dots & a_{12} \\ a_{13} & a_{14} & \dots & \dots & \dots & a_{18} \\ a_{19} & a_{20} & \dots & \dots & \dots & a_{24} \end{array}\right] \middle| \, a_i \in \right.$

    $S([0, 48))$; $1 \leq i \leq 24$, max$\}$ be the subset interval semigroup super matrix.

    Study questions (i) to (iv) of problem 12 for this L.

19. Let W = $\left\{\left[\begin{array}{ccc|ccc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ a_7 & a_8 & \dots & \dots & \dots & a_{12} \\ a_{13} & a_{14} & \dots & \dots & \dots & a_{18} \\ \hline a_{19} & a_{20} & \dots & \dots & \dots & a_{24} \\ a_{25} & a_{26} & \dots & \dots & \dots & a_{30} \\ \hline a_{31} & a_{32} & \dots & \dots & \dots & a_{36} \\ \hline a_{37} & a_{38} & \dots & \dots & \dots & a_{42} \end{array}\right] \middle| \, a_i \in \right.$

    $S([0, 9))$; $1 \leq i \leq 42$, max$\}$ be the subset interval matrix semigroup.

    Study questions (i) to (iv) of problem 12 for this W.

20. Let S = $\{S([0, 23)), \times\}$ be the interval subset semigroup.

(i) Prove S has infinite number of zero divisors.
(ii) Can S have S-zero divisors?
(iii) Prove S has only finite number of units and idempotents.
(iv) Can S have S-units?
(v) Can S have S-idempotents?
(vi) Can S have S-ideals?
(vii) Can S have S-subsemigroup?
(viii) Can S have ideals which are not S-ideals?
(ix) Obtain any other special feature enjoyed by S.

21. Let M = {S([0, 124)), ×} be the interval subset semigroup.

Study questions (i) to (ix) of problem 20 for this M.

22. Let L = $\left\{\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{18} \end{bmatrix} \middle| a_i \in S([0, 12)); \times\right\}$ be the subset interval column matrix semigroup.

Study questions (i) to (ix) of problem 20 for this L.

23. Let B = $\left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & \cdots & \cdots & \cdots & a_{10} \\ a_{11} & \cdots & \cdots & \cdots & a_{15} \\ a_{16} & \cdots & \cdots & \cdots & a_{20} \end{bmatrix} \middle| a_i \in S([0, 127)); 1 \leq i \leq 20, \times\right\}$ be the subset interval matrix semigroup.

Study questions (i) to (ix) of problem 20 for this B.

24. Let M = $\left\{\left[\begin{array}{cc|ccc} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & \cdots & \cdots & \cdots & a_{10} \\ \hline a_{11} & \cdots & \cdots & \cdots & a_{15} \\ a_{16} & \cdots & \cdots & \cdots & a_{20} \\ a_{21} & \cdots & \cdots & \cdots & a_{25} \\ \hline a_{26} & \cdots & \cdots & \cdots & a_{30} \\ \hline a_{31} & \cdots & \cdots & \cdots & a_{35} \\ a_{36} & \cdots & \cdots & \cdots & a_{40} \\ a_{41} & \cdots & \cdots & \cdots & a_{45} \end{array}\right]\right| a_i \in S([0, 28));$

$1 \leq i \leq 45, \times\}$ be the subset interval semigroup super matrix.

Study questions (i) to (ix) of problem 20 for this M.

25. Let B = {S ([0, 15)), min, max} be the subset interval semiring.

(i) Prove o(B) = ∞.
(ii) Prove B has subsemirings of order one, two, three and so on.
(iii) Can B have ideals of finite order?
(iv) Is it possible to define S-subsemirings and S-ideals on B?
(v) Is B a S-semiring?
(vi) Can B have zero divisors?
(vii) Can the concept of S-idempotents be defined in B?
(viii) Can B have finite lattices?

26. Let S = {S ([0, 18)), min, max} be the subset interval semiring.

Study questions (i) to (viii) of problem 25 for this S.

27. Let A = $\{(a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mid a_i \in S([0, 43))$, min, max$\}$ be the subset interval row matrix semiring.

    Study questions (i) to (viii) of problem 25 for this A.

28. Let B = $\left\{\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{18} \end{bmatrix} \middle|\, a_i \in S([0, 44));\ 1 \leq i \leq 18,\ \text{min, min}\right\}$

    be the subset interval column semiring.

    Study questions (i) to (viii) of problem 25 for this B.

29. Let M = $\left\{\begin{bmatrix} a_1 & a_2 & a_3 & \ldots & a_9 \\ a_{10} & a_{11} & a_{12} & \ldots & a_{18} \\ a_{19} & a_{20} & a_{21} & \ldots & a_{27} \\ a_{28} & a_{29} & a_{30} & \ldots & a_{36} \\ a_{37} & a_{38} & a_{39} & \ldots & a_{45} \end{bmatrix} \middle|\, a_i \in S([0, 43));\right.$

    $1 \leq i \leq 45$, min, max$\}$ be the interval subset matrix semiring.

    Study questions (i) to (viii) of problem 25 for this M.

30. Let S = $\left\{\left[\begin{array}{ccc|c} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ \hline a_{13} & a_{14} & a_{15} & a_{16} \end{array}\right] \middle|\, a_i \in S([0, 48));\right.$

$1 \leq i \leq 16$, min, max} be the subset interval super matrix semiring.

Study questions (i) to (viii) of problem 25 for this S.

31. Obtain any other special and interesting properties enjoyed subset interval semiring.

32. Obtain all special properties enjoyed by subset interval pseudo ring.

33. Let B = {S([0,23)), +, ×} be the subset interval pseudo ring.

    (i) Prove o(B) = ∞.
    (ii) Prove distributive laws are not true in general in P.
    (iii) Find pseudo ideals of P.
    (iv) Is every ideal of B is of infinite order?
    (v) Can P have S-pseudo ideals?
    (vi) Can P have S-pseudo subrings which are not S-ideals?
    (vii) Can P have S-zero divisors?
    (viii) Can P have units which are not S-units?
    (ix) Can P have S-idempotents?
    (x) Can S have semi idempotents?

34. Let M ={S([0, 48)), +, ×} be the pseudo interval subset ring.

    Study questions (i) to (x) of problem 33 for this M.

35. Let $W = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{18} \end{bmatrix} \middle| \right.$ $a_i \in S([0, 7)); 1 \leq i \leq 18, +, \times_n\}$ be the

    pseudo subset interval column matrix semiring.

Study questions (i) to (x) of problem 33 for this W.

36. Let V = $\{(a_1, a_2, \ldots, a_9) \mid a_i \in S([0, 23)),\ 1 \leq i \leq 9, +, \times\}$ be the subset interval row matrix pseudo ring.

Study questions (i) to (x) of problem 33 for this V.

37. Let T = $\left\{\begin{bmatrix} a_1 & a_2 & a_3 & \ldots & a_9 \\ a_{10} & a_{11} & a_{12} & \ldots & a_{18} \\ a_{19} & a_{20} & a_{21} & \ldots & a_{27} \end{bmatrix} \middle| a_i \in S([0, 43));\right.$

$1 \leq i \leq 27, +, \times_n\}$ be the pseudo subset interval matrix ring.

Study questions (i) to (x) of problem 33 for this T.

38. Let S = $\{(a_1, a_2, \ldots, a_9) \mid a_i \in S(C[0, 3)); 1 \leq i \leq 9, \times\}$ be the subset interval matrix semigroup.

Study questions (i) to (x) of problem 33 for this S.

39. Let W = $\left\{\begin{bmatrix} a_1 & a_2 & a_3 & \ldots & a_9 \\ a_{10} & a_{11} & a_{12} & \ldots & a_{18} \end{bmatrix} \middle| a_i \in\right.$

$S(\langle[0, 9) \cup I\rangle); 1 \leq i \leq 18, \times_n\}$ be the subset interval finite complex modulo integer neutrosophic semigroup.

Study questions (i) to (x) of problem 33 for this W.

40. Let B = $\left\{\begin{bmatrix} a_1 & a_2 & \ldots & a_{10} \\ a_{11} & a_{12} & \ldots & a_{20} \\ a_{21} & a_{22} & \ldots & a_{30} \\ \vdots & \vdots & \vdots & \vdots \\ a_{81} & a_{82} & \ldots & a_{90} \end{bmatrix} \middle| a_i \in S(\langle[0, 18) \cup I\rangle);\right.$

$1 \leq i \leq 90, \times_n\}$ be the subset interval neutrosophic matrix semigroup.

Study questions (i) to (x) of problem 33 for this B.

41. Let P $= \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \end{bmatrix} \right| a_i \in S(C\langle[0, 7) \cup I\rangle);$

$1 \leq i \leq 12$, min} be the subset interval semigroup of finite complex modulo integer matrices.

Study questions (i) to (x) of problem 33 for this P.

42. Let W $= \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \right| a_i \in S(\langle[0, 45) \cup I\rangle);$

$1 \leq i \leq 16$, min} be the subset interval neutrosophic matrix semigroup.

Study questions (i) to (ix) of problem 20 for this W.

43. Let S = $\left\{\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \\ a_{11} & a_{12} \\ a_{13} & a_{14} \\ a_{15} & a_{16} \\ a_{17} & a_{18} \\ a_{19} & a_{20} \\ a_{21} & a_{22} \end{bmatrix} \middle| a_i \in S(C\langle[0, 3) \cup I\rangle); 1 \leq i \leq 22,\right.$

min} be the subset interval super matrix finite complex modulo integer semigroup.

Study questions (i) to (ix) of problem 20 for this S.

44. Let M = $\{(a_1, a_2, a_3, \ldots, a_9) \mid a_i \in S(C([0, 11)), 1 \leq i \leq 9,$ max} be the subset interval row matrix finite complex modulo integer semigroup.

Study questions (i) to (ix) of problem 20 for this M.

45. Let T = $\left\{\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_9 \end{bmatrix} \middle| a_i \in S(C\langle[0, 7) \cup I\rangle); 1 \leq i \leq 9, \max \right\}$

be the subset interval finite complex modulo integer neutrosophic semigroup.

Study questions (i) to (ix) of problem 20 for this T.

46. Let L = $\left\{\left.\begin{bmatrix} a_1 & a_2 & a_3 & \dots & a_7 \\ a_8 & a_9 & a_{10} & \dots & a_{14} \\ a_{15} & a_{16} & a_{17} & \dots & a_{21} \\ a_{22} & a_{23} & a_{24} & \dots & a_{28} \\ a_{29} & a_{30} & a_{31} & \dots & a_{35} \end{bmatrix} \right| a_i \in\right.$ $S(\langle[0, 15) \cup I\rangle)$; $1 \leq i \leq 35$, max} be the subset interval neutrosophic semigroup.

    Study questions (i) to (ix) of problem 20 for this L.

47. Let W = $\{(a_1, a_2, \dots, a_{18}) \mid a_i \in S(C\langle[0, 11) \cup I\rangle)$; $1 \leq i \leq 18$, $\times\}$ be the subset interval finite complex modulo integer semigroup.

    (i) Study questions (i) to (ix) of problem 20 for this W.

    (ii) Can W have finite subsemigroup?

    (iii) Prove every ideal of W is of infinite order.

48. Let S = $\left\{\left.\left[\begin{array}{cc|ccc} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ \hline a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} & a_{19} & a_{20} \end{array}\right] \right| a_i \in\right.$ $S(\langle[0, 19) \cup I\rangle)$; $1 \leq i \leq 20$, $\times_n\}$ be the subset interval finite neutrosophic semigroup.

    Study questions (i) to (iii) of problem 47 for this S.

49. Let $V = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \end{bmatrix} \middle| a_i \in S(\langle C[0, 12) \cup I \rangle); 1 \leq i \leq 8, \times_n \right\}$ be

    the subset interval finite complex integer neutrosophic semigroup.

    Study questions (i) to (iii) of problem 47 for this V.

50. Let M = $\{(a_1, a_2, \ldots, a_{18}) \mid a_i \in S([0, 9)); 1 \leq i \leq 18, \min, \max\}$ be the subset interval finite complex modulo integer semiring of row matrices.

    (i) Prove $o(M) = \infty$.

    (ii) Show $P = \{\{(0, \ldots, 0), x\} \mid x \in M\}$ is a subsemiring of order two.

    (iii) Is P an ideal?

    (iv) Can ideals in M be of finite order?

    (v) Show every element in M is idempotent.

    (vi) Can we have order three, order four, …, order n (n > 1) subsemirings?

    (vii) Obtain any other special property associated with this type of semirings.

51. Let

$$V = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \\ a_{11} & a_{12} \\ a_{13} & a_{14} \end{bmatrix} \right| a_i \in S(\langle[0, 11) \cup I\rangle); 1 \leq i \leq 14,$$

min, max} be the subset interval neutrosophic matrix semiring.

Study questions (i) to (vii) of problem 50 for this V.

52. Let

$$S = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \end{bmatrix} \right| a_i \in$$

$S(\langle C[0, 15) \cup I\rangle)$; $1 \leq i \leq 15$, min, max} be the subset interval complex modulo integer neutrosophic semigroup.

Study questions (i) to (vii) of problem 50 for this S.

53. Let W = $\{(a_1, a_2, \ldots, a_{29}) \mid a_i \in S(C[0, 19)); 1 \leq i \leq 29, \cup, \cap\}$ be the finite complex modulo integer interval semiring.

Study questions (i) to (vii) of problem 50 for this W.

54. Let B = $\left\{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \end{bmatrix} \middle| a_i \in S(\langle[0, 19) \cup I\rangle); 1 \leq i \leq 7, \cup, \cap\right\}$ be

    the subset interval neutrosophic column matrix semiring.

    Study questions (i) to (vii) of problem 50 for this B.

55. Let S = $\left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & \cdots & \cdots & \cdots & a_{10} \\ a_{11} & \cdots & \cdots & \cdots & a_{15} \\ a_{16} & \cdots & \cdots & \cdots & a_{20} \\ a_{21} & \cdots & \cdots & \cdots & a_{25} \\ a_{26} & \cdots & \cdots & \cdots & a_{30} \end{bmatrix} \middle| a_i \in\right.$

    $S(C\langle[0, 23) \cup I\rangle); 1 \leq i \leq 30, \cup, \cap\}$ be the subset interval finite complex modulo integer neutrosophic matrix semiring.

    Study questions (i) to (vii) of problem 50 for this S.

56. Compare semirings with min, max operations with semirings under $\cup$ and $\cap$ operations.

    Are they identical or distinct? Justify.

57. Let S = $\{(a_1, a_2, \ldots, a_{10}) \mid a_i \in S(C([0, 5)); 1 \leq i \leq 10$, min, $\times\}$ be the finite complex modulo integer pseudo semiring.

    (i) Study questions (i) to (vii) of problem 50 for this S.

    (ii) Show we can have subsemiring to be both a pseudo filter and pseudo ideal.

    (iii) Show S has infinite number of zero divisors.

    (iv) Can S be a S-pseudo semiring?

    (v) Can S have S-zero divisors?

    (vi) Can S have S-idempotents with respect to $\times$?

    (vii) Can S have S-units?

58. Let

$$P = \left\{ \left[\begin{array}{cc|cccc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ a_7 & \cdots & \cdots & \cdots & \cdots & a_{12} \\ \hline a_{13} & \cdots & \cdots & \cdots & \cdots & a_{18} \\ a_{19} & \cdots & \cdots & \cdots & \cdots & a_{24} \\ a_{25} & \cdots & \cdots & \cdots & \cdots & a_{30} \end{array} \right] \middle| \, a_i \in \right.$$

$S(\langle[0, 13) \cup I\rangle)$; $1 \leq i \leq 30, +, \times_n\}$ be the subset interval neutrosophic super matrix pseudo ring.

Study questions (i) to (vii) of problem 57 for this P.

59. Let $S = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \end{bmatrix} \middle| a_i \in S(\langle [0, 12) \cup I \rangle); 1 \leq i \leq 7, +, \times_n \right\}$ be

the subset interval neutrosophic column matrix finite complex modulo integer pseudo ring.

Study questions (i) to (vii) of problem 57 for this S.

60. Let V = $\{(a_1, a_2, \ldots, a_9) \mid a_i \in S(C([0, 13)); 1 \leq i \leq 9, +, \times\}$ be the finite complex integer matrix pseudo ring.

(i) Prove $a \times (b + c) \neq a \times b + a \times c$ for some a, b, c $\in$ V.

(ii) Prove V is commutative.

(iii) Can V have zero divisors?

(iv) Can V have S-zero divisors?

(v) Can V have S-units?

(vi) Can V have S-idempotents?

(vii) Is V a S-peudo ring?

(viii) Can V have S-pseudo subrings?

(ix) Can V have S-pseudo ideals?

(x) Can V have S-subrings which are not S-ideals?

(xi) Give some interesting features enjoyed by these S-pseudo rings.

61. Let S =

$$\left\{\left.\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 \\ a_{10} & a_{11} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{18} \\ a_{19} & a_{20} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{27} \\ a_{28} & a_{29} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{36} \end{bmatrix}\right| a_i \in\right.$$

$S(\langle[0, 15) \cup I\rangle)$; $1 \leq i \leq 36, +, \times_n\}$ be the subset interval neutrosophic matrix pseudo ring.

Study questions (i) to (xi) of problem 60 for this S.

62. Let S = $\left\{\left.\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_9 \end{bmatrix}\right| a_i \in S(C(\langle[0, 12) \cup I\rangle)); 1 \leq i \leq 9, +, \times_n\right\}$

be the subset interval finite complex modulo integer neutrosophic pseudo ring.

Study questions (i) to (ix) of problem 60 for this S.

63. Let M = $\left\{\left.\begin{bmatrix} a_1 & a_2 & \cdots & a_{10} \\ a_{11} & a_{12} & \cdots & a_{20} \end{bmatrix}\right| a_i \in S(C(\langle[0, 8) \cup I\rangle));\right.$

$1 \leq i \leq 20, +, \times_n\}$ be the subset interval finite complex modulo integer matrix pseudo ring.

Study questions (i) to (ix) of problem 60 for this M.

64. Obtain any of the special features enjoyed by subset interval pseudo rings (built using S(C([0, n)) or S(⟨[0, n)∪ I⟩) or S S(C⟨[0, n)∪ I⟩).

65. Let $V = \{(a_1, \ldots, a_9) \mid a_i \in S(C([0, 17)); 1 \le i \le 9, +\}$ be the subset interval finite complex modulo integer vector space over Z_{17}.

    (i) Is V finite dimensional?
    (ii) Find a basis of V over Z_{17}.
    (iii) Can V have several basis?
    (iv) Write V as a direct sum of subspace.
    (v) Enumerate some special feature enjoyed by V.
    (vi) Find $\text{Hom}_{Z_{17}}(V, V) = S$.
    (vii) What is the algebraic structure enjoyed by S?
    (viii) Is S a subset interval vector space over Z_{17}?
    (ix) If S a vector space over Z_{17}, what is the dimension of S over Z_{17}?
    (x) Is it possible to define inner product on V?
    (xi) Can we define linear functionals on V and get V* (V* dual space of V).?
    (xii) Will $V^* \cong V$?

66. Let $V = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_9 \end{bmatrix} \middle| \, a_i \in S(\langle [0, 19) \cup I \rangle); 1 \le i \le 9, + \right\}$ be a subset interval neutrosophic column matrix vector space over the field Z_{19}.

    Study questions (i) to (xii) of problem 65 for this V.

67. Let $S = \{(a_1, \ldots, a_7) \mid a_i \in SS([0, 5)), +\}$ be the super subset interval group of row matrices under +.

(i) Prove o(S) = ∞ and is commutative.

(ii) Prove S has subgroups of finite order.

(iii) Can S have subgroups of infinite order?

(iv) Write S as a direct sum of subgroups.

(v) Give any other property enjoyed by S.

68. Let B = $\left\{\left.\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{20} \end{bmatrix}\right| a_i \in S(C[0, 12));\ 1 \leq i \leq 20, +\right\}$ be the super interval subset column matrix group of finite complex modulo integers.

Study questions (i) to (v) of problem 67 for this B.

69. Let M = $\left\{\left.\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & \ldots & a_9 \\ a_{10} & a_{11} & a_{12} & a_{13} & \ldots & a_{18} \\ a_{19} & a_{20} & a_{21} & a_{22} & \ldots & a_{27} \\ a_{28} & a_{29} & a_{30} & a_{31} & \ldots & a_{36} \end{bmatrix}\right| a_i \in S(\langle[0, 14) \cup I\rangle);\ 1 \leq i \leq 36, +\right\}$ be the super subset interval matrix group of neutrosophic values.

Study questions (i) to (v) of problem 67 for this M.

70. Let T = $\left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & \dots & a_9 \\ a_{10} & a_{11} & a_{12} & a_{13} & \dots & a_{18} \end{bmatrix} \middle| a_i \in\right.$ $S(C\langle[0, 18) \cup I\rangle)$; $1 \leq i \leq 18, +\}$ be the super subset interval finite complex modulo integer neutrosophic group.

Study questions (i) to (v) of problem 67 for this T.

71. Let S = $\left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & \dots & \dots & \dots & a_{10} \\ a_{11} & \dots & \dots & \dots & a_{15} \\ a_{16} & \dots & \dots & \dots & a_{20} \end{bmatrix} \middle| a_i \in SS([0, 18));\right.$ $1 \leq i \leq 20$, min} be the super subset interval matrix semigroup.

(i) Prove o(S) = ∞ and S is commutative.
(ii) Prove every element in S is an idempotent and a subsemigroup.
(iii) Can S have subsemigroups of all order?
(vi) Can S be a Smarandache semigroup?
(v) Is it possible for a subsemigroup to be a S-subsemigroup?
(vi) Can S have S-ideals?
(vii) Can S have S-idempotents?

72. Let B = $\{(a_1, \dots, a_9) \mid a_i \in SS(C([0, 9))$, $1 \leq i \leq 9$, min} be the super subset interval finite complex modulo integer semigroup.

Study questions (i) to (vii) of problem 71 for this B.

73. Study problems 71 and 72 when min is replaced by maximum.

74. Let W = $\left\{\left.\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{12} \end{bmatrix} \right| a_i \in SS(\langle[0, 14) \cup I\rangle);\ 1 \le i \le 12, \max\right\}$

be the super subset interval neutrosophic column matrix semigroup.

Study questions (i) to (vii) of problem 71 for this W.

75. Let V = $\left\{\left.\begin{bmatrix} a_1 & a_2 & \dots & a_{15} \\ a_{16} & a_{17} & \dots & a_{30} \end{bmatrix} \right| a_i \in S(C(\langle[0, 19) \cup I\rangle);\right.$

$1 \le i \le 30$, min (or max)} be the super subset interval finite complex modulo integer neutrosophic semigroup.

Study questions (i) to (vii) of problem 71 for this V.

76. Let M = $\left\{\left.\left[\begin{array}{cc|ccc|cc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ a_8 & \dots & \dots & \dots & \dots & \dots & a_{14} \\ \hline a_{15} & \dots & \dots & \dots & \dots & \dots & a_{21} \\ a_{22} & \dots & \dots & \dots & \dots & \dots & a_{28} \\ a_{29} & \dots & \dots & \dots & \dots & \dots & a_{35} \\ \hline a_{36} & \dots & \dots & \dots & \dots & \dots & a_{42} \\ a_{43} & \dots & \dots & \dots & \dots & \dots & a_{45} \end{array}\right] \right| a_i \in\right.$

SS([0, 12)); $1 \le i \le 19$, $\cap$} be the super subset interval semigroup of super matrices.

(i) Study questions (i) to (vii) of problem 71 for this M.
(ii) Does M enjoy any other special feature?

77. Study problem 76 when in M '$\cap$' is replaced by the operation '$\cup$'.

78. Let V = $\{(a_1, a_2, a_3, \ldots, a_{12}) \mid a_i \in SS(C([0, 27)), 1 \leq i \leq 12, \cap\}$ be the super subset interval finite complex modulo integer semigroup.

    Study questions (i) to (vii) of problem 71 for this V.

79. Study problem 78 when '$\cap$' is replaced by $\cup$.

80. Let $S = \left\{ \begin{bmatrix} a_1 & a_2 & \ldots & a_{10} \\ a_{11} & a_{12} & \ldots & a_{20} \\ a_{21} & a_{22} & \ldots & a_{30} \\ a_{31} & a_{33} & \ldots & a_{40} \\ a_{41} & a_{42} & \ldots & a_{50} \end{bmatrix} \middle| a_i \in SS(\langle [0, 18) \cup I \rangle); \right.$

    $1 \leq i \leq 50, \cup\}$ be the super subset interval neutrosophic matrix semigroup.

    Study questions (i) to (vii) of problem 71 for this S.

81. Study problem 80 replacing '$\cup$' by '$\cap$' in S.

82. Let M = $\{(a_1, a_2, a_3, \ldots, a_9) \mid a_i \in SS(C(\langle [0, 15) \cup I \rangle)), 1 \leq i \leq 9, \cap$ (or $\cup$)$\}$ be the interval super subset of finite complex modulo integer neutrosophic row matrix semigroup.

    Study questions (i) to (vii) of problem 71 for this M.

83. Let S = $\left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & \dots & \dots & \dots & a_{10} \\ a_{11} & \dots & \dots & \dots & a_{15} \end{bmatrix} \middle| a_i \in SS([0, 24));\right.$

$1 \leq i \leq 15, \times_n\}$ be the super subset interval matrix semigroup.

(i) Show o(S) = ∞.
(ii) Show S has infinite number of zero divisors.
(iii) Show S has only finite number of units and idempotents.
(iv) Can S have S-zero divisors?
(v) Can S have S-units?
(vi) Can S have S-idempotents?
(vii) Is S a Smarandache semigroup?
(viii) Can S have S-subsemigroups?
(ix) Can S have S-ideals?
(x) Specify any other special feature enjoyed by S.
(xi) Can S have ideals or S-ideals or finite order?

84. Let S = $\left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & \dots & \dots & \dots & a_{10} \\ a_{11} & \dots & \dots & \dots & a_{15} \\ a_{16} & \dots & \dots & \dots & a_{20} \end{bmatrix} \middle| a_i \in SS([0, 43));\right.$

$1 \leq i \leq 20, \cup, \cap\}$ be the super subset interval matrix semiring.

(i) Prove o(S) = ∞.
(ii) Prove S has finite order subsemiring.
(iii) Can S have finite order ideals?
(iv) Can S have S-subsemirings of finite order?
(v) Can S have S-idelas of finite order?
(vi) Can S have filters?
(vii) Can ideals in S be a filters in S?

85. Let V = $\left\{\left.\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ a_7 & \cdots & \cdots & \cdots & \cdots & a_{12} \\ a_{13} & \cdots & \cdots & \cdots & \cdots & a_{18} \\ a_{19} & \cdots & \cdots & \cdots & \cdots & a_{24} \end{bmatrix}\right| a_i \in\right.$

SS(C[0, 12))); $1 \leq i \leq 24$, $\cup$, $\cap$ } be the super subset interval complex modulo integer semiring.

Study questions (i) to (vii) of problem 84 for this V.

86. Let W = $\left\{\left.\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_9 \end{bmatrix}\right| a_i \in \text{SS}(\langle[0, 11) \cup I\rangle); 1 \leq i \leq 9, \cap, \cup\right\}$

be the super subset interval neutrosophic column matrix semiring.

Study questions (i) to (vii) of problem 84 for this W.

87. Let S = $\left\{\left.\left[\begin{array}{ccc|c|cc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ a_8 & \cdots & \cdots & \cdots & \cdots & \cdots & a_{14} \\ a_{15} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{21} \\ \hline a_{22} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{28} \\ \hline a_{29} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{35} \\ a_{36} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{42} \\ a_{43} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{45} \\ \hline a_{46} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{56} \end{array}\right]\right| a_i \in\right.$

SS(C$\langle$[0, 23) $\cup$ I$\rangle$); $1 \leq i \leq 56$, $\cup$, $\cap$} be the super subset interval finite complex modulo integer neutrosophic super matrix semiring.

Study questions (i) to (vii) of problem 84 for this S.

88. Let P = $\{(a_1, a_2, a_3, \ldots, a_{11}) \mid a_i \in SS([0, 42)), 1 \leq i \leq 11,$ min, max$\}$ be the super subset interval row matrix semiring.

(i) Study questions (i) to (vii) of problem 84 for this P.

(ii) In P if max and min is replaced by $\cup$ and $\cap$ compare them.

89. Let $M = \left\{ \begin{bmatrix} a_1 & a_2 & \ldots & a_{10} \\ a_{11} & a_{12} & \ldots & a_{20} \\ a_{21} & a_{22} & \ldots & a_{30} \end{bmatrix} \middle| a_i \in SS(C([0, 15)));\right.$

$1 \leq i \leq 30$, min, max$\}$ be the subset super interval complex integer modulo semiring.

(i) Study questions (i) to (vii) of problem 84 for this M.

90. Let $L = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ \vdots & \vdots & \vdots \\ a_{34} & a_{35} & a_{36} \end{bmatrix} \middle| a_i \in SS(\langle [0, 37) \cup I \rangle);\right.$

$1 \leq i \leq 36$, max, min$\}$ be the subset super interval neutrosophic semiring.

Study questions (i) to (vii) of problem 84 for this L.

91. Let $M = \left\{ \begin{bmatrix} a_1 & a_2 & \ldots & a_9 \\ a_{10} & a_{11} & \ldots & a_{18} \\ a_{19} & a_{20} & \ldots & a_{27} \end{bmatrix} \middle| a_i \in SS(C(\langle[0, 24)) \cup \right.$ $\left. I\rangle)); 1 \leq i \leq 27, \min, \max \right\}$ be the super subset interval complex modulo integer neutrosophic semiring.

    (i) Study questions (i) to (vii) of problem 84 for this M.

92. Compare M when min, max operations in M is replaced by $\cap$ and $\cup$.

93. Let $S = \{(a_1, a_2, a_3, \ldots, a_{12}) \mid a_i \in SS([0, 24)), 1 \leq i \leq 12,$ $\min, \times\}$ be the super subset interval pseudo semiring.

    (i) Show $o(S) = \infty$.

    (ii) Prove S has infinite number of zero divisors.

    (iii) Can S have subsemiring which is both a pseudo filter and pseudo ideal?

    (iv) Can S have S-subsemirings?

    (v) Can S have infinite number of idempotents and units?

    (vi) Can S have S-units?

    (vii) Can S have S-idempotents?

    (viii) Find any other special and distinct features enjoyed by the pseudo semiring S.

94. Let $S = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_9 \end{bmatrix} \middle| a_i \in SS(C[0, 12)); 1 \leq i \leq 9, \min, \times \right\}$ be the super subset interval finite complex modulo integer pseudo semiring.

Study questions (i) to (viii) of problem 93 for this S.

95. Let $B = \left\{ \left[\begin{array}{cc|ccc|c} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & \dots & \dots & \dots & a_{10} \\ \hline a_{11} & \dots & \dots & \dots & a_{15} \\ a_{16} & \dots & \dots & \dots & a_{20} \\ a_{21} & \dots & \dots & \dots & a_{25} \\ \hline a_{26} & \dots & \dots & \dots & a_{30} \end{array}\right] \middle| a_i \in SS(\langle [0, 24) \cup I \rangle); 1 \leq i \leq 30, \times_n, \min \right\}$ be the super subset interval neutrosophic super matrix pseudo semiring.

Study questions (i) to (viii) of problem 93 for this B.

96. Let $L = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & \dots & a_9 \\ a_{10} & a_{11} & a_{12} & a_{13} & \dots & a_{18} \\ a_{19} & a_{20} & a_{21} & a_{22} & \dots & a_{27} \\ a_{28} & a_{29} & a_{30} & a_{31} & \dots & a_{36} \\ a_{37} & a_{38} & a_{39} & a_{40} & \dots & a_{45} \end{bmatrix} \middle| a_i \in SS(C(\langle [0, 15) \cup I \rangle)); 1 \leq i \leq 45, \min, \times_n \right\}$ be the super subset interval finite complex modulo integer neutrosophic pseudo semiring.

Study questions (i) to (viii) of problem 93 for this L.

97. Let S = $\{(a_1, a_2, a_3, \ldots, a_{15}) \mid a_i \in SS([0, 18)), 1 \leq i \leq 15, +, \times\}$ be the super subset interval pseudo ring.

(i) Show o(S) = ∞.
(ii) Prove S has infinite number of zero divisor?
(iii) Can S have infinite number of S-idempotents?
(iv) Can S have infinite number of units?
(v) Can S have S units?
(vi) Can S have S-ideals?
(vii) Can S have S-subrings which are not S-ideals.
(viii) Prove S has no ideals of finite order.
(ix) Find any stricking feature enjoyed by S.

98. Let B = $\left\{\left[\begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ a_5 & \ldots & \ldots & a_8 \\ a_9 & \ldots & \ldots & a_{12} \\ a_{13} & \ldots & \ldots & a_{16} \\ a_{17} & \ldots & \ldots & a_{20} \\ a_{21} & \ldots & \ldots & a_{24} \end{array}\right] \middle| a_i \in SS(C(\langle[0, 25)); \right.$

$1 \leq i \leq 24, +, \times_n\}$ be the super interval subset complex modulo integer semiring.

Study questions (i) to (ix) of problem 97 for this B.

99. Let T = $\left\{\left(\begin{array}{c|ccc|cc|c|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \\ a_9 & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{16} \end{array}\right) \middle| a_i \right.$

$\in SS(\langle[0, 27)) \cup I\rangle); 1 \leq i \leq 16, +, \times_n\}$ be the super interval neutrosophic super row matrix pseudo ring.

Study questions (i) to (ix) of problem 97 for this T.

100. Let $M = \left\{\left[\begin{array}{cccc} a_1 & a_2 & \ldots & a_{10} \\ a_{12} & a_{12} & \ldots & a_{20} \\ a_{21} & a_{22} & \ldots & a_{30} \\ a_{31} & a_{32} & \ldots & a_{40} \\ a_{41} & a_{42} & \ldots & a_{50} \end{array}\right] \right| a_i \in$

$\in SS(C\langle[0, 15) \cup I\rangle)$; $1 \leq i \leq 50, +, \times_n\}$ be the super subset interval finite complex modulo integer neutrosophic matrix semiring.

Study questions (i) to (ix) of problem 97 for this M.

101. Let $M = \{(a_1, a_2, \ldots, a_{15}) \mid a_i \in SS([0, 29))$; $1 \leq i \leq 15, +\}$ be the super subset interval vector space over $F = Z_{29}$.

(i) What is the dimension of M over F?

(ii) Find a basis of M over F.

(iii) Can M have several basis over F?

(iv) Find subspaces of M of finite dimension over F.

(v) Write M as a direct sum of subspaces.

(vi) Find Hom(M, M) = S.
Is S a super subset interval vector space over F?

(vii) Describe any other special feature enjoyed by M.

102. Let S = $\left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & \cdots & \cdots & \cdots & a_{10} \\ a_{11} & \cdots & \cdots & \cdots & a_{15} \\ a_{16} & \cdots & \cdots & \cdots & a_{20} \\ a_{21} & \cdots & \cdots & \cdots & a_{25} \end{bmatrix} \middle| \, a_i \in SS(C[0, 23));\right.$

$1 \leq i \leq 25, + \}$ be the super subset interval vector space of finite complex modulo integer matrix over the field Z_{23}.

Study questions (i) to (vii) of problem 101 for this S.

103. If S in problem 102 is defined over the S-ring $C(\langle Z_{23} \cup I\rangle)$ and renamed as V.

Study questions (i) to (vii) of problem 101 for this V.

104. If S in problem 102 is defined over the S-ring $C(Z_{23})$ and renamed as W.

Study questions (i) to (vii) of problem 101 for that W.

105. If S in problem 102 is defined over the S-pseudo ring $R = \{[0, 23), +, \times\}$ and S be the renamed as B.

Study questions (i) to (vii) of problem 101 for this S- pseudo vector space B over R.

106. Let W = $\left\{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} \middle| \, a_i \in SS(\langle [0, 7) \cup I\rangle); 1 \leq i \leq 6, + \right\}$

be a super subset interval vector space defined the field $F = Z_7$.

(i) Study questions (i) to (vii) of problem 101 for this W.

(ii) If W in problem 105 is defined over the S-ring $P = \langle Z_7 \cup I\rangle$ that is the changed and denoted by W_1, W_1 the S-vector space over P.

Study questions (i) to (vii) of problem 101 for this W_1.

107. Let $S = \{(a_1, a_2, a_3, a_4, a_5) \mid a_i \in SS([0, 43)); 1 \leq i \leq 5, +\}$ be the strong pseudo super subset interval vector space over the super strong subset interval pseudo ring $R = \{SS([0, 43)), +, \times\}$.

Study questions (i) to (vii) of problem 101 for this S.

108. Let $W = \left\{\left[\begin{array}{c} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{array}\right] \middle| a_i \in SS(C[0, 3)); 1 \leq i \leq 6, +, \times_n\right\}$

be the super subset interval S-super strong pseudo linear algebra over the S-super strong pseudo ring $R = \{SS(C([0, 3)), +, \times\}$.

(i) Study Hom (W, W).

(ii) Find a basis of W over R.

(iii) Can W be made into a pseudo inner product space?

(iv) Find W^*.

(v) Is $W \cong W^*$ over R?

(vi) Is W finite dimensional over R?

(vii) Can spectral theorem be derived for W?

(viii) Write W as a direct sum and obtain projections and related results.

109. Find some special features associated with S-pseudo super subset interval super strong linear algebras over $R = \{SS([0, n)), +, \times\}$ (or $\{SS(C[0, n)), +, \times\}$ or $\{SS(\langle[0, n) \cup I\rangle), +, \times\}$ or $\{SS(C(\langle[0, n) \cup I\rangle)), +, \times\}$).

110. Prove on S-super interval subset vector spaces over $C(Z_p)$ one cannot define pseudo inner product.

111. Prove on S-super subset interval vector spaces over the S-ring $R = \{S([0, n)), \times, +\}$ one cannot define pseudo inner product.

112. Obtain a necessary and sufficient condition for a S-super interval subset strong vector space to be a pseudo inner product space.

113. Let V be S-super subset interval vector space over a S- pseudo ring, $R = \{S([0, n)), +, \times\}$.

Study all the special features enjoyed by V.

Differentiate V from the usual vector space.

114. Suppose $V = \left\{\left.\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix}\right| a_i \in\right.$

$SS(C(\langle[0, 23) \cup I\rangle)), 1 \leq i \leq 16, +, \times_n\}$ and

$W = \left\{\left.\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ \vdots & \vdots \\ a_{19} & a_{20} \end{bmatrix}\right| a_i \in SS(C(\langle[0, 23) \cup I\rangle)), 1 \leq i \leq 20,\right.$

$+, \times_n\}$ be two S-super strong subset interval finite complex modulo integer neutrosophic vector spaces over the S-super strong subset interval pseudo ring;
$R = \{ SS(C(\langle[0, 23) \cup I\rangle)), +, \times\}$.

(i) Find $T = \{Hom_R(V, W)\}$.

(ii) What is the algebraic structure enjoyed by T?

(iii) Find $B = \{Hom_R(V, V)$ and $D = \{Hom_R(W, W)\}$.

(iv) Is $B \cong D$ as spaces?

(v) Is V and W inner product spaces?

(vi) Find V^* and W^* and show $V \cong V^*$ and $W \cong W^*$.

(vii) Can V^* and W^* be related through f, where $f : V \to W$ is a S-linear transformation of V to W?

115. Obtain any special feature enjoyed by S-subset super interval vector space over the S-ring $C(Z_p)$ or $(\langle Z_p \cup I\rangle$ or $C(\langle Z_p \cup I\rangle))$.

116. Obtain some special features enjoyed by the S-pseudo special super subset interval linear algebras (vector spaces) defined over the S-pseudo subset ring; $S([0, n))$ (or $S(C([0, n))$ or $SS(\langle[0, n) \cup I\rangle)$ or $S(C(\langle[0, n) \cup I\rangle))$.

Chapter Three

SPECIAL TYPE OF TOPOLOGICAL SPACES BUILT USING [0, n)

In this chapter we define, develop and describe several special type of topological spaces built using subsets of [0, n) or intervals of [0, n).

We use the notations from chapter two. For sake of completeness we just recall the notations given in chapter two.

S([0, n)) = {All elements of the form [x, y) or [x,y] x, y $\in$ [0, n); that is intervals and naturals class of intervals as x > y is also permitted}.

SS([0, n)) = {Collection of all subsets from the elements in S[0, n)}.

We define topology of S([0, n)) or SS([0, n)) using the basic algebraic structure enjoyed by the sets.

DEFINITION 3.1: *Let*
*S = {Collection of all subsets of the semigroup P = {[0, n), *}. S is a subset interval semigroup under the operation *. We define three types of interval topological spaces called the*

ordinary or usual or standard interval topological spaces of the interval semigroup P.

Let $S' = S \cup \{\phi\}$.

$T_o = \{S', \cup, \cap\}$ is the ordinary or usual or standard type of subset interval topological space of the super subset interval semigroup S.

$T_\cup = \{S, \cup, *\}$ and $T_\cap = \{S', \cap, *\}$ will be known as the special type of super subset interval topological spaces of the super subset interval semigroup $S = \{\{SS([0, n)), *\}$.

We will first illustrate this situation by some examples.

Example 3.1: Let S = {Collection of all subsets of the semigroup, P = {[0, 7), min}} be the subset super interval semigroup; S = {SS([0, 7)), min}.

$T_o = \{S' = S \cup \{\phi\}, \cup, \cap\}$ is the subset interval super special ordinary topological space of S.

Now $T_\cup^{\min} = \{\{SS([0, 7)), \cup, \min\}$
$= \{S, \cup, \min\}$ is a super interval special topological space of the super subset interval semigroup.

We can also have a super interval subset topological space $T_\cap^{\min} = \{SS([0, 7)) \cup \{\phi\}, \cap, \min\}$.

We will show by illustration that all the three spaces are different.

Let A = {2.07, 5.21, 0.71, 5, 6} and
B = {0, 2.07, 5.21, 0.9, 1.2, 4, 3, 2.4, 0.8} $\in$ SS([0, 7)).

Let us consider A, B $\in T_o$.

$A \cap B = \{2.07, 5.21\}$ and

$A \cup B = \{0, 2.07, 5.21, 0.9, 1.2, 4, 3, 2.4, 0.8, 0.71, 5, 6\}$ are in T_o.

Let $A, B \in T_{\cup}^{\min}$;

$\min \{A, B\} = \{0, 2.07, 0.9, 1.2, 0.8, 5.21, 4, 3, 2.4, 0.7\}$ and
$A \cup B = \{0, 2.07, 5.21, 0.9, 1.2, 4, 3, 2.4, 0.8, 0.7, 5.6\}$ are in $T_{\cup}^{\min}$.

Let $A, B \in T_{\cap}^{\min}$.
$\min \{A, B\} = \{0, 2.07, 0.9, 1.2, 0.8, 5.21, 4, 3, 2.4, 0.7\}$ and

$A \cap B = \{2.07, 5.21\} \in T_{\cap}^{\min}$.

Thus T_o, $T_{\cup}^{\min}$ and $T_{\cap}^{\min}$ are three distinct topological spaces of infinite order.

Thus using a semigroup we get three super subset interval topological spaces.

Example 3.2: Let S = {Collection of all subsets from the interval semigroup of P = {[0, 12), min} = {SS([0, 12)), min}.

We have 3 super subset interval special topological spaces.

$$T_o = \{S' = S \cup \{\phi\}, \cup, \cap\},$$

$$T_{\cup}^{\min} = \{S, \cup, \min\}$$
and $T_{\cap}^{\min} = \{S, \cap, \min\}$.

All the three spaces are of infinite order.

However all the three spaces have subspaces of finite order (By finite order we mean the space has finite number of elements in them).

Example 3.3: Let S ={Collection of all subsets from the interval semigroup; P = {[0, 24), max}}

= {SS([0, 24), max}.

We can define three different types of special topological spaces.

$$T_o = \{S' = S \cup \{\phi\}, \cup, \cap\}$$

$$T_{\cup}^{\max} = \{S, \cup, \max\} \text{ and}$$

$T_{\cap}^{\max} = \{S', \cap, \max\}$ are the three super subset interval special topological spaces. All the three spaces are of infinite order and distinct.

Let A = {9.3, 8.74, 14.5, 20.7, 0.32, 6.637, 8.1001, 5.026, 4.4, 7.07, 10.37, 4.003}

and B = {9.3, 8.1001, 10.37, 21.5, 14.32, 6.003,10.31, 8.34} $\in T_o$;

A $\cup$ B = {9.3, 8.74, 14.5, 20.7, 0.32, 6.637, 8.1001, 8.34, 5.026, 4.4, 7.07,10.37, 4.003, 21.5, 14, 32, 6.003,10.31} … I

and A $\cap$ B = {9.3, 8.1001, 10.37} are in T_o.

Let A, B $\in T_{\cup}^{\max}$.

max{A,B}= {9.3, 10.37, 21.5, 14.32, 10.31, 8.74,14.5, 20.7, 6.637, 6.003, 8.1001, 7.07, 8.34} and

A $\cup$ B is given in I.

Both max{A, B} and A $\cup$ B $\in T_{\cup}^{\max}$ are different from each other.

Likewise we can find $T_{\cap}^{max}$ and it is clear that all the three spaces are distinct however the cardinality is the same.

Further all the three spaces are disconnected.

Example 3.4: Let S = {Collection of all subsets of the interval semigroup, P = {[0, 5), max}} = {SS([0, 5)), max}.

On S we can define three distinct super special subset interval special topological spaces. S is of infinite cardinality viz T_o, $T_{\cup}^{max}$ and $T_{\cap}^{max}$.

All the three of them are distinct. Infact S has subtopological spaces of finite order.

For take A = {All subsets from the set Z_5}, S_o, $S_{\cup}^{max}$ and $S_{\cap}^{max}$ be three distinct subspaces all of them are of finite order 2^5.

Infact based on this example we just have the following theorem.

THEOREM 3.1: *Let S = {Collection of all subsets from the interval semigroup P = {[0, n), min} or (P = {[0, n), max})} = {SS([0, n)), min (or max)} be the super subset interval semigroup.*

(1) On S we have three distinct super special subset interval topological spaces viz T_o, $T_{\cup}^{min}$ and $T_{\cap}^{min}$ (or T_o, $T_{\cup}^{max}$ and $T_{\cap}^{max}$) of infinite cardinality.

(2) T_o, $T_{\cup}^{max}$ and $T_{\cap}^{max}$ (or T_o, $T_{\cup}^{min}$ and $T_{\cap}^{min}$) has infinite number of super special subset interval topological subspaces of finite order.

(3) *T_o, $T_{\cup}^{max}$ and $T_{\cap}^{max}$ (or T_o, $T_{\cup}^{min}$ and $T_{\cap}^{min}$) also have subspaces of infinite order.*

(4) *T_o, $T_{\cup}^{max}$ and $T_{\cap}^{max}$ (or T_o, $T_{\cup}^{min}$ and $T_{\cap}^{min}$) are not connected.*

The proof is direct and hence left as an exercise to the reader.

Example 3.5: Let S = {Collection of all subsets from the interval semigroup P = {[0, 12), min}} = {SS([0, 12)), min} the super subset interval semigroup.

Let A = {0.4, 6, 8. 10.3, 5.7, 4.3} $\in T_o$, ($T_{\cup}^{\min}$ or $T_{\cap}^{\min}$).

M_o = {A, ϕ} $\subseteq T_o$ is a subspace of the super subset interval topological space.

M_1 = {A, ϕ, $\cup$, min} $\subseteq T_{\cup}^{\min}$ is a subspace of cardinality two.

M_2 = {A, ϕ, $\cap$, min} $\subseteq T_{\cap}^{\min}$ is a subspace of $T_{\cap}^{\min}$.

Let L = {A = {4.3, 10.7, 2.8, 4.03} and
B = {9.3, 6.7, 4, 10, 5.3}} $\subseteq T_o$ ($T_{\cup}^{\min}$ or $T_{\cap}^{\min}$).

We now complete L in T_o

L^C = {A, B, A $\cup$ B, A $\cap$ B}
= {A, B, ϕ, {4.3, 10.7, 2.8, 4.03, 9.3, 6.7, 4, 10, 5.3}} $\subseteq T_o$.

L^C is a topological subspace of cardinality four. The basis for L^C is {A, B}.

Consider L as a subset of $T_{\cup}^{\min}$.

To find the completion of L with respect to the operations min and $\cup$.

Let W be the completion of L; W ={A, B, {4.3, 4, 9.3 6.7, 5.3, 10, 2.8, 4.03}, {4.3, 10.7, 2.8, 4.03, 9.3,6.7, 4, 10, 5.3}} $\subseteq$

$T_{\cup}^{\min}$ is the completion of the set L and W is a topological subspace of $T_{\cup}^{\min}$.

A basis for W is {A, B}.

Consider $L \subseteq T_{\cap}^{\min}$, L is only a subset of $T_{\cap}^{\min}$. To find the completion L to a subspace of $T_{\cap}^{\min}$.

Let V be the completion of the set L under the operation min and $\cap$.

$V = \{A, B, A \cap B = \phi, \{4.3, 2.8, 4.03, 9.3, 6.7, 10, 4, 5.3\}\} \subseteq T_{\cap}^{\min}$ is a subspace of $T_{\cap}^{\min}$ and a basis of V is {A, B}.

Thus if L is a set with two elements which are not singleton sets. $A \neq B$ ($A \subseteq B$ or $B \subseteq A$ or $A \neq \phi$ or $A = S$} then the completion of L in a topological subspace has cardinality to be always four.

Let $L = \{A = \{3.7\}$ and $B = \{9.17\}\} \subseteq T_o$.

The completion of L into a topological space of T_o is as follow:

$L^C = \{A, B, \{3.7, 9.17\}$.

The completion of L as a topological subspace of $T_{\cup}^{\min}$ denoted by $W = \{A, B, \phi, \{3.7, 9.17\}\}$.

The completion of L as a topological subspace of $T_{\cap}^{\min}$ denoted by $V = \{A, B, \phi\}$.

Thus $|V| = 3$ and $|W| = 3$ but $|L^C| = 4$. However the basis is {A, B}.

Let $L = \{A = \{3.4\}, B = \{6\}$ and $C = \{2.4\}\}$ be a subset of T_o. The completion of L into a topological space denoted by

L^C = {A, B, C, ϕ, {3.4, 6}, {2.4, 6}, {2.4, 3.4}, {3.4, 6, 2.4}} $\subseteq T_o$ is a subspace of cardinality 8. Let L $\subseteq T_{\cup}^{\min}$. To find the completion of L say W as a subspace of $T_{\cup}^{\min}$.

W = {A, B, C, {3.4, 2.4}, {2.4, 6}, {3.4,6}, {2.4, 3.4, 6}} $\subseteq T_{\cup}^{\min}$ is a subspace of cardinality 7.

Let V be the completion of L as a subspace of $T_{\cap}^{\min}$.

V ={A, B, C, ϕ} $\subseteq T_{\cap}^{\min}$ is a subspace of order 4. Thus we get several spaces of different orders.

Thus we leave the following problem as an open conjecture.

Conjecture 3.1: Let T_o be a super subset interval special topological space.

Let L = {$A_1, A_2,\ldots, A_n$} be a collection of singleton subsets of T_o; $|A_i| = i$ and $A_i \neq A_j$ if $i \neq j$.

If |L| = n, what is the cardinality of L^C in T_o?

Conjecture 3.2: Let L = {$A_1, A_2,\ldots, A_n$} be n distinct singleton sets in $T_{\cup}^{\min}$ ($T_{\cup}^{\max}$). Let V be the completion of L in $T_{\cup}^{\min}$ that the subtopological space generated by L under min and $\cup$ operation (max and $\cup$ operation).

If |L| = n find o(V).

Conjecture 3.3: Let L = {$A_1, A_2,\ldots, A_n$} be n distinct singleton sets in $T_{\cap}^{\min}$ (or $T_{\cup}^{\max}$). Let W be the completion of L in $T_{\cap}^{\min}$ (or $T_{\cup}^{\max}$) that is subspace generated by L.

If |L| = n find order of W.

Compare the orders of L^C, W and V of T_o, $T_{\cap}^{min}$ and $T_{\cup}^{max}$ respectively.

Example 3.6: Let S = {Collection of all subsets of the semigroup P ={[0, 13), min} ={SS ([0, 13)), min} the super subset interval semigroup. $T_o = \{S' = S \cup \{\phi\}, \cup, \cap\}$, $T_{\cup}^{min}$ = {S, min, ∪} and $T_{\cap}^{min}$ = {S′, min, ∩} are all super subset interval special topological spaces of infinite cardinality and they have infinite basis.

However T_o, $T_{\cup}^{min}$ and $T_{\cap}^{min}$ have subspaces of finite cardinality.

Now we can get topological spaces of semigroups built using SS([0, n)), which will be illustrated by the following examples.

Example 3.7: Let S = {Collection of all subsets from the semigroup P ={$(a_1, a_2, \ldots, a_5)$ | $a_i \in [0, 9)$, max}} be the super interval special subset interval row matrix semigroup.

$T_o = \{S' = S \cup \{\phi\}, \cup, \cap\}$,

$T_{\cup}^{max}$ = {S, max, ∪} and $T_{\cap}^{max}$ = {S′, max, ∩} are the super subset interval row matrix special topological spaces.

Let A ={(0.3, 4, 2, 5, 0), (6, 0, 7, 3.2, 4.5), (0.9, 2, 0, 1, 3.1)} and B ={(0.9, 2, 0, 1, 3.1), (2.7, 3.5, 4.8, 5.6, 7.2)} $\in T_o$.

A ∩ B = {(0.9, 2, 0, 1, 3.1)} = D and

A ∪ B = {(0.3, 4, 2, 5, 0), (6, 0, 7, 3.2, 4.5), (0.9, 2, 0, 1, 3.1), (2.7, 3.5, 4.8,5.6, 7.2)} = C $\in T_o$.

Further P = {ϕ, A, B, D, A ∪ B = C} is a subspace of T_o.

Let A, B $\in T_{\cup}^{max}$.

Now max{A, B} = {(0.9, 4, 2, 5, 3.1), (6, 2, 7, 3.2, 4.5) (0.9, 2, 0, 1, 3.1), (2.7, 4, 4.8, 5.6, 7.2), (6, 3.5, 7, 5.6, 7.2), (2.7, 3.5, 4.8, 5.6, 7.2)} = E.

M = {⟨ A, B, E, C⟩} ∈ $T_{\cap}^{\max}$ generates a subspaces of $T_{\cap}^{\max}$. When matrices are used this is the way operations are performed.

Example 3.8: Let

$$P = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_9 \end{bmatrix} \middle| \; a_i \in [0, 14), 1 \leq i \leq 9, \min \right\}$$

be the subset interval semigroup.

Let S_1 = {Collection of all subsets of P, min} be the super interval subset special subsemigroup under min operation.

$T_o = \{S' = S \cup \{\phi\}, \cup, \cap\}$.

$T_{\cup}^{\min}$ = {S, min, ∪} and $T_{\cap}^{\min}$ = {S′, min, ∩} are the three types of super subset interval special topological spaces of column matrices.

Now it is pertinent to keep on record S and S_1 in examples 3.7 and 3.8 are different from

$R_1 = \{(a_1, a_2, a_3, a_4, a_5) \mid a_i \in SS([0, 9)), \min\}$ and

$$R_2 = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_9 \end{bmatrix} \middle| \; a_i \in SS([0, 14)), 1 \leq i \leq 9, \min \right\} \text{ respectively.}$$

a_i's in R_2 and R_1 are subsets from SS([0,14)) where as elements in S and S_1 are matrix subsets.

That is x = ({0.3, 4.5, 7}. (2, 4.6, 3.1}, {0}, {2.3, 8.1}, {8.101}) $\notin$ S but x $\in$ R where as y = {(0.3, 0, 1, 0.7, 8.1), (6.1, 4.5, 3.2, 4.01, 0.71)} $\in$ S and y $\notin$ R.

The same is true in case of S_1 and R_1. Thus S_1 and R_1 are different. Similarly S_1 and R_2 are different.

Example 3.9: Let M = $\{(a_1, a_2, a_3) | a_i \in [0, 19), 1 \leq i \leq 3, \min\}$ be the subset interval semigroup.

B = {Collection of all subsets from M, min} is the special super subset interval semigroup. $M_1 = \{(b_1, b_2, b_3) | b_i \in [0, 19), 1 \leq i \leq 3, \min\}$ be the super subset interval semigroup.

Clearly M and M_1 are different.

Example 3.10: Let

$$M = \left\{\left[\begin{matrix} a_1 & a_2 \\ a_3 & a_4 \\ \vdots & \vdots \\ a_{29} & a_{30} \end{matrix}\right] \middle| \, a_i \in [0, 25), 1 \leq i \leq 30, \min\right\}$$

be the interval subset semigroup.

Let P = {Collection of all subsets from M, min}. P is also a super interval subset semigroup.

$$\text{Let } B = \left\{\left[\begin{matrix} a_1 & a_{16} \\ a_2 & a_{17} \\ \vdots & \vdots \\ a_{15} & a_{16} \end{matrix}\right] \middle| \, a_i \in SS([0, 25)), 1 \leq i \leq 30, \min\right\}$$

be the super subset interval semigroup. Clearly B and P are distinct.

So B gives way to topological spaces which are different from the three topological spaces given by P.

Example 3.11: Let S = {Collection of all subsets from the semigroup

$$P = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & \dots & \dots & a_8 \\ a_9 & \dots & \dots & a_{12} \\ a_{13} & \dots & \dots & a_{16} \end{bmatrix} \middle| \; a_i \in [0, 27), 1 \le i \le 16, \min \right\}\}$$

be the super special subset interval semigroup.

$$S_1 = \left\{ \begin{bmatrix} b_1 & b_2 & b_3 & b_4 \\ b_5 & \dots & \dots & b_8 \\ b_9 & \dots & \dots & b_{12} \\ b_{13} & \dots & \dots & b_{16} \end{bmatrix} \middle| \; b_i \in SS([0, 27)), 1 \le i \le 16, \min \right\}$$

be the super subset interval semigroup.

Clearly S and S_1 are distinct.

Using S we get three super subset interval special topological spaces T_o, $T_{\cup}^{\min}$ and $T_{\cap}^{\min}$.

Similarly for S_1 we get three super subset interval topological spaces P_o, $P_{\cup}^{\min}$ and $P_{\cap}^{\min}$ and $T_o \ne P_o$, $P_{\cup}^{\min} \ne T_{\cup}^{\min}$ and $T_{\cap}^{\min} \ne P_{\cap}^{\min}$.

Studying these two spaces comparing them happens to be an interesting research work.

Further {SS([0, n), min} = {Collection of all subsets of the semigroup P = {[0, n), min}} as super special subset interval semigroups. Only when the matrices are built both the spaces are distinct.

THEOREM 3.2: *Let M = {Collection of all subsets from subset interval semigroup P = {m × n matrices with entries from [0, s); min}} be the subset interval semigroup.*

Let M_1 = {m × n matrices with entries from SS([0, s)), min} be the super subset interval semigroup.

(i) $M_1 \neq M$ (that is M and M_1 are distinct as semigroups).

(ii) The 3 topological spaces associated with M is different from the three topological spaces associated with M_1.

The proof is left as an exercise to the reader.

In the theorem if 'min' operation is replaced by max still the result holds good.

Example 3.12: Let S_1 = {Collection of all subsets from the subset interval matrix semigroup.

$$P_1 = \left\{ \left[\begin{array}{cccc} a_1 & a_2 & \ldots & a_9 \\ a_{10} & a_{11} & \ldots & a_{18} \\ a_{19} & a_{20} & \ldots & a_{27} \\ a_{28} & a_{29} & \ldots & a_{36} \end{array} \right] \right| a_i \in ([0, 11)),$$

$$1 \leq i \leq 36, \max\}, \max\}$$

be the super subset interval semigroup.

$$P_2 = \left\{ \left[\begin{matrix} b_1 & b_2 & \ldots & b_9 \\ b_{10} & b_{11} & \ldots & b_{18} \\ b_{19} & b_{20} & \ldots & b_{27} \\ b_{28} & b_{29} & \ldots & b_{36} \end{matrix} \right] \middle| \; b_i \in SS([0, 11)), \right.$$

$$1 \leq i \leq 36, \max\}$$

be the super subset interval matrix semigroup.

Clearly P_1 and P_2 are distinct. Hence the three topological spaces associated with them are also distinct.

Example 3.13: Let S_1 = {Collection of all subsets from the interval semigroup matrix

$$P_1 = \left\{ \left[\begin{matrix} a_1 & a_2 & \ldots & a_{18} \\ a_{19} & a_{20} & \ldots & a_{36} \end{matrix} \right] \middle| \; a_i \in ([0, 12)), 1 \leq i \leq 36, \times\}\right\}$$

be the super subset interval matrix semigroup.

$$S_2 = \left\{ \left[\begin{matrix} b_1 & b_2 & \ldots & b_{18} \\ b_{19} & b_{20} & \ldots & b_{36} \end{matrix} \right] \middle| \; b_i \in SS([0, 12)), 1 \leq i \leq 36, \times \right\}$$

be the super subset interval matrix semigroup.

Clearly S_1 and S_2 are distinct. Both the semigroups have infinite number of zero divisors.

We have super subset interval special topological spaces associated with them viz;

$$T_o = \{ S_1' = S_1 \cup \{\phi\}, \cup, \cap\},$$

$$T_{\cup}^{\times} = \{S_1, \times, \cup\} \text{ and } T_{\cap}^{\times} = \{ S_1', \cap, \times\}.$$

The three super subset interval special topological spaces associated with S_2 is $P_o = \{S_2' = S_2 \cup \{\phi\}, \cup, \cap\}$,

$P_{\cup}^{\times} = \{S_2, \times, \cup\}$ and $P_{\cap}^{\times} = \{S_2', \cap, \times\}$ are three distinct super special subset interval topological spaces different from T_o, $T_{\cup}^{\times}$ and $T_{\cap}^{\times}$ associated with S_1.

The topological spaces $P_{\cup}^{\times}$, $P_{\cap}^{\times}$, $T_{\cup}^{\times}$ and $T_{\cap}^{\times}$ has infinite number of zero divisors.

Example 3.14: Let L = {Collection of all subsets from the subset interval semigroup $P = \{(a_1, a_2, a_3) \mid a_i \in [0, 4), 1 \leq i \leq 3, \times\}\}$ be the super subset interval semigroup.

Let x = {(0.3, 1.4, 2), (2.4, 2, 0.6), (3, 2, 1), (0, 3.4, 0.2)} and
y = {(0.7, 2, 1), (0.4, 0.5, 3), (2, 1, 0)} $\in$ L.

x $\times$ y = (0.21, 2.8, 2), (1.68, 0,0.6) (2.1, 0, 1), (0, 2.8,0.2), (0.12,0.70, 2), (0.96, 1, 1.8), (1.2,1, 3), (0, 1.70, 0.6), (0.6, 1.4, 0), (0.8,2, 0), (2, 2, 0), (0, 3.4,0)}.

This is the way operation is performed on L.

$T_o = (L', \cup, \cap)$ where $L' = L \cup \{0\}$.

$T_{\cup}^{\times} = \{L, \cup, \times\}$ and $T_{\cap}^{\times} = \{L', \cap, \times\}$ are the three subset super interval special row matrix topological spaces associated with L.

Let x = {(0.3, 0, 1), (0.8, 1, 3.2), (2.01, 0,2)} and
y = {(2.4, 2.11, 3), (0.7, 0.9, 0)} $\in T_o$,

x $\cup$ y = (0.3, 0, 1), (0.8, 1, 3.2), (2.01, 0,2), (1.407, 0, 0), (2.4, 2.11, 3), (0.56, 0.9, 0)} and x $\cap$ y = $\phi \in T_o$.

Let x, y $\in T_{\cup}^{\times}$;

$x \times y = \{(0.72, 0, 3), (1.92, 2.11, 1.6), (0.824, 0, 2), (1.407, 0, 0), (0.21, 0, 0.56), (0.56, 0.9, 0)\}$ --- D

and $x \cup y = \{(0.3, 0.1), (0.8, 1,3.2), (2.01, 0,2), (2.4, 2.11, 3), (0.7, 0.9, 0)\} \in T_{\cup}^{\times}$.

Let $x, y \in T_{\cap}^{\times}$;

$x \cap y = \phi$ and $x \times y = D \in T_{\cap}^{\times}$.

All the three spaces are distinct.

Infact $T_{\cap}^{\times}$ and $T_{\cup}^{\times}$ has non trivial zero divisors.

Let $x = \{(0, 2.1, 1.4), (0, 0, 3.2), (0, 3, 2.144), (0, 2, 0.5)\}$

and $y = \{(2, 0, 0), (0.115, 0, 0), (2.0004, 0, 0), (0.712, 0,0), (0.03015, 0, 0), (1.0052, 0, 0)\} \in T_{\cup}^{\times}$ (or $T_{\cap}^{\times}$).

Clearly $x \times y = \{(0, 0, 0)\}$.

Example 3.15: Let

$$S = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \middle| \; a_i \in SS([0, 5), 1 \leq i \leq 3, \times\} \right.$$

be the super subset interval semiring.

$T_o = \{S' = S \cup \{\phi\}, \cap, \cup\}$, $T_{\cup}^{\times} = \{S, \cup, \times\}$ and $T_{\cap}^{\times} = \{S', \cap, \times\}$ are the three super subset interval special topological spaces.

Let x = $\begin{bmatrix} \{0.3, 4, 2, 1.2\} \\ \{0.5, 3, 0.6\} \\ \{3, 2, 1, 4\} \end{bmatrix}$

and y = $\begin{bmatrix} \{0.7, 0.8, 4, 2\} \\ \{3, 2, 0.6, 0.9, 0.5\} \\ \{1, 2, 3, 0.1, 0.01, 0.002\} \end{bmatrix} \in T_o$.

$x \cup y = \begin{bmatrix} \{0.3, 4, 2, 1.2, 0.7, 0.8\} \\ \{0.5, 3, 0.6, 0.9, 2\} \\ \{1, 2, 3, 0.1, 0.01, 0.002, 4\} \end{bmatrix}$ = p and

$x \cap y = \begin{bmatrix} \{4, 2\} \\ \{3, 0.5, 0.6\} \\ \{1, 2, 3\} \end{bmatrix} = q \in T_o$.

Now we take x, y $\in T_{\cup}^{\times}$ and $x \cup y$, $x \cap y \in T_{\cup}^{\times}$.

$$x \times y = \begin{bmatrix} \begin{matrix} \{0.21, 2.8, 1.4, 0.84, 0.24 \\ 3.2, 1.6, 0.96, 1.2, 1, 3, 4.8, \\ 0.6, 3, 4, 2.4\} \end{matrix} \\ \\ \begin{matrix} \{1.5, 4, 1, 1.8, 1, 0, 1.2, \\ 3, 4.5, 2.7, 0.4, 0.25, 1.5, 0.3\} \end{matrix} \\ \\ \begin{matrix} \{3, 2, 1, 4, 0.3, 0.2, 0.1, 0.4, \\ 0.03, 0.02, 0.01, 0.04, 0.006, \\ 0.004, 0.002, 0.008\} \end{matrix} \end{bmatrix} = r.$$

Consider x, y $\in T_{\cap}^{\times}$. $x \cap y = q$ and $x \times y = r \in T_{\cap}^{\times}$.

This is the way operations are performed on topological spaces.

We see $T_{\cap}^{\times}$ and $T_{\cup}^{\times}$ have infinite number of zero divisors.

Example 3.16: Let S ={Collection of all subsets from the subset interval semigroup

$$P = \left\{\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \middle| a_i \in [0, 6), 1 \leq i \leq 4, \times\}, \times_n\right\}$$

be the super subset interval semigroup. P has zero divisors.

T_o, $T_{\cup}^{\times}$ and $T_{\cap}^{\times}$ are three distinct topological spaces such that $T_{\cup}^{\times}$ and $T_{\cap}^{\times}$ has zero divisors.

$$\text{Let } x = \left\{\begin{bmatrix} 0.7 & 4 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 5 & 2.1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 3.612 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 4.123 & 0 \\ 0 & 0 \end{bmatrix}\right\} \text{ and}$$

$$y = \left\{\begin{bmatrix} 0 & 0 \\ 0.8 & 1.5 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0.63 & 1.25 \end{bmatrix},\right.$$

$$\left.\begin{bmatrix} 0 & 0 \\ 0 & 3.52 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 4 & 5 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 3.121 & 1.3121 \end{bmatrix}\right\} \in T_{\cap}^{\times} \text{ (or } T_{\cup}^{\times}\text{)}$$

$$\text{Clearly } x \times_n y = \left\{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\right\}.$$

Hence $T_{\cap}^{\times}$ and $T_{\cup}^{\times}$ have infinite number of zero divisors.

Infact $T_{\cup}^{\times}$ and $T_{\cap}^{\times}$ are known as super subset interval special topological spaces with zero divisors.

Example 3.17: Let

$$S = \left\{\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \middle| \, a_i \in S([0, 6)),\ 1 \le i \le 4, \times_n\right\}$$

be the super subset interval semigroup. T_o, $T_{\cup}^{\times_n}$ and $T_{\cap}^{\times_n}$ be the super topological subset interval spaces associated with S.

T_o, $T_{\cup}^{\times_n}$ and $T_{\cap}^{\times_n}$ have infinite number of zero divisors.

Let A = $\begin{bmatrix} \{0.3, 0.7, 2, 4\} & \{0\} \\ \{3, 0\} & \{2, 4\} \end{bmatrix}$ and

B = $\begin{bmatrix} \{0\} & \{3.8, 4.5, 0.6, 3.2\} \\ \{2, 4\} & \{3, 0\} \end{bmatrix} \in T_{\cup}^{\times_n}$ (or $T_{\cap}^{\times_n}$).

Clearly A $\times_n$ B = $\begin{bmatrix} \{0\} & \{0\} \\ \{0\} & \{0\} \end{bmatrix}$.

Thus we can have infinite number of zero divisors.

Let

$$A = \begin{bmatrix} \{0\} & \{2, 4, 3.2, 4.2, 5.7, 3.771, 1.205\} \\ \{0.7, 4.32, 5.1, 3.11\} & \{2.001, 4.112, 5.0116, 3.8\} \end{bmatrix}$$

and

$$B = \begin{bmatrix} \{0.31, 3.715, 4.32, 5.117, 2.12\} & \{0\} \\ \{0\} & \{0\} \end{bmatrix} \in T_{\cup}^{\times_n} \text{ (or } T_{\cap}^{\times_n}\text{)}.$$

$$\text{Clearly } A \times_n B = \begin{bmatrix} \{0\} & \{0\} \\ \{0\} & \{0\} \end{bmatrix}.$$

THEOREM 3.3: *Let S = {Collection of all subsets from the subset interval semigroup P = {$m \times n$ matrices with entries from [0, s), $\times_n$}} be the super subset interval matrix semigroup. Associated with S are three super subset interval special topological spaces T_o, $T_{\cup}^{\times_n}$ (or $T_{\cap}^{\times_n}$).*

$T_{\cup}^{\times_n}$ and $T_{\cap}^{\times_n}$ are zero divisor super special subset interval topological spaces.

Proof is left as an exercise to the reader.

THEOREM 3.4: *Let*

S_1 = {$m \times n$ matrices with entries from SS([0, s)), $\times_n$} be the super subset special interval semigroup. Associated with this S_1 we have three distinct topological spaces T_o, $T_{\cup}^{\times_n}$ and $T_{\cap}^{\times_n}$.

$T_{\cup}^{\times_n}$ and $T_{\cap}^{\times_n}$ are super subset special interval zero divisor topological spaces.

Proof is direct hence left as an exercise to the reader.

Remark : The S and S_1 in Theorem 3.3 and 3.4 are distinct hence the 3 super subset interval special topological space are also distinct.

All other properties can be had as from [ ].

Next we define, describe and develop the topological spaces associated with a super subset interval semiring.

DEFINITION 3.2: *Let R = {Collection of all subsets from the pseudo ring S ={[0, n), +, $\times$}}. R is a subset pseudo semiring of type I.*

Example 3.18: Let S = {Collection of all subsets of the subset interval pseudo ring R = {[0, 12), +. ×}}. S is a semiring of type I under the operation $\cup$ and $\cap$.

Example 3.19: Let M = {Collection of all subsets of the subset interval pseudo ring R = {[0, 19), +. ×}}. M is a semiring of type I.

Example 3.20: Let M = {Collection of all subsets of the subset interval pseudo ring; R = $\{(a_1, a_2, \ldots, a_6) \mid a_i \in [0, 16), 1 \leq i \leq 6, +, \times\}\}$ be the subset semiring of type I.

Example 3.21: Let

$$M = \left\{ \left. \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_9 \end{bmatrix} \right| a_i \in [0, 15), 1 \leq i \leq 9, +, \times_n \right\}$$

be the subset interval pseudo ring. M = {Collection of all subset from the subset interval pseudo ring} be the subset semiring of type I.

Example 3.22: Let S = {Collection of all subsets from the subset interval pseudo ring R} be the subset interval semiring of type I where

$$R = \left\{ \left. \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \end{bmatrix} \right| a_i \in [0, 18), 1 \leq i \leq 12, +, \times_n \right\}\}.$$

Now we built six types of topological spaces we call all of them as pseudo super subset interval topological spaces.

$T_o = \{S' = S \cup \{\phi\}, \cup, \cap\}$,

$T_{\cup}^{+} = \{S, \cup, +\}$,

$T_{\cap}^{+} = \{S', \cap, +\}$,

$T_{\cup}^{\times} = \{S, \times, \cup\}$,

$T_{\cap}^{\times} = \{S', \times, \cap\}$ and $T_s = \{S, +, \times\}$.

It is clear these super subset interval pseudo topological spaces are built using a pseudo subset interval ring.

We will show by an example that all the six spaces are distinct.

Example 3.23: Let S = {Collection of all subsets from the subset interval pseudo ring R ={[0, 3), +, ×}} be the subset semiring of type I.

$T_o = \{S' = S \cup \{\phi\}, \cup, \cap\}$ be the ordinary pseudo subset semiring interval topological space.

$T_{\cup}^{+} = \{S, +, \cup\}$ be the pseudo subset interval semiring topological space.

$T_{\cap}^{+} = \{S', +, \cap\}$, $T_{\cup}^{\times} = \{S, \times, \cup\}$, $T_{\cap}^{\times} = \{S', \times, \cap\}$, $T_s = \{S, +, \times\}$ be pseudo subset interval semiring topological spaces.

We first show all the six spaces are different.

Let A = {0.3, 2.1, 0.7, 0.1, 0.03,0.07, 0.06} and
B = {0.3, 2.1, 0.005, 0.001, 0.0002, 0.2, 0.6} $\in T_o$.

$A \cap B$ ={0.3, 2.1} = C and

$A \cup B$ = {0.3, 2.1,0.7, 0.1, 0.03, 0.07,0.06, 0.005, 0.001, 0.0002, 0.2,0.6} = $D \in T_o$.

Now let for the same A, B $\in T_{\cup}^{+}$ we find;

A + B = {0.6, 2.4, 1, 0.305, 0.301, 0.3002, 0.5, 0.9, 1.2, 2.105, 2.101, 2.1002,2.3, 2.7, 0.1, 2.8, 0.705, 0.701, 0.7002, 1.3, 0.4, 2.2, 0.105, 0.101, 0.1002, 0.3, 0.7,. 0.33, 2.13, 0.031, 0.0302, 0.065, 0.035, 0.075, 0.23, 0.63, 0.37, 2.17, 0.075, 0.071, 0.0702, 0.27, 0.67, 0.36, 2.16, 0.065, 0.061, 0.0602, 0.26, 0.66} = E and

$A \cup B = D \in T_{\cup}^{+}$.

Clearly $T_{\cup}^{+}$ and T_o are different as pseudo subset interval topological spaces.

Consider A, B $\in T_{\cap}^{+}$.

Clearly $A \cap B = C$ and $A + B = E \in T_{\cap}^{+}$.

$T_{\cap}^{+}$ is different as a pseudo subset interval topological space from T_o and $T_{\cup}^{+}$.

Let A, B $\in T_{\cup}^{\times}$ we find;

$A \times B$ = {0.09, 0.63, 0.21, 0.03, 0.009, 0.021, 0.018, 1.41, 0.0105, 0.0021, 0.00042, 0.42, 1.26, 1.47, 0.0035, 0.0007, 0.00014, 0.14, 0.42, 0.03, 0.21, 0.0005, 0.0001, 0.00002, 0.02, 0.06, 0.009, 0.063, 0.00015, 0.00003, 0.000006, 0.006, 0.018, 0.021, 0.147, 0.014, 0.042, 0.018, 0.021, 0.147, 0.00035, 0.0007, 0.000014, , 0..014, 0.042, 0.018, 0.126,0.00030, 0.00006, 0.000012, 0.012, 0.036} = F;

$A \cup B = D$ and

F, D $\in T_{\cup}^{\times}$.

Clearly $T_{\cup}^{\times}$ is different from T_o, $T_{\cup}^{+}$ and $T_{\cap}^{+}$.

Let A, B $\in T_{\cap}^{\times}$

$A \times B = F$ and $A \cap B = C$, so F, C $\in T_{\cap}^{\times}$.

Clearly $T_{\cap}^{\times}$ is different from $T_{\cup}^{\times}$, $T_{\cup}^{+}$, $T_{\cap}^{+}$ and T_o.

Let A, B $\in T_s$.

A + B = E and A $\times$ B = F $\in T_s$. T_s is different from T_o, $T_{\cup}^{\times}$, $T_{\cup}^{+}$, $T_{\cap}^{+}$ and $T_{\cap}^{\times}$.

Thus all the pseudo subset special interval semiring topological spaces are different.

Example 3.24: Let R = {SS([0, 3)), +, $\times$} be the super subset interval pseudo ring we see R and S in example 3.23 are identical and for the same A, B given in example 3.23 the same solution is true.

Inview of all these we have the following theorem.

THEOREM 3.5: *Let S = {SS([0, n)), +, ×} be the super subset special interval pseudo ring and R ={all subsets from [0, n), +, ×} be the pseudo subset interval ring:*

(i) R = S.

(ii) All the 6 pseudo ring topological spaces on R are identical with S.

Proof is direct and hence left as an exercise to the reader.

Example 3.25: Let

$$S = \left\{ \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \middle| \; a_1, a_2 \in SS([0, 9)), +, \times_n \right\}$$

be the super subset interval pseudo matrix.

R = {Collection of all subsets from the subset pseudo interval ring

$$P = \left\{ \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \middle| \; a_1, a_2 \in [0, 9), +, \times_n \right\} \}.$$

We see S and R are distinct. Further the semiring topological spaces associated with them are also distinct.

Let T_o, $T_{\cup}^{+}$, $T_{\cap}^{+}$, $T_{\cup}^{\times}$, $T_{\cap}^{\times}$ and T_s be the super subset pseudo interval semiring topological spaces associated with S.

Let S_o, $S_{\cup}^{+}$, $S_{\cap}^{+}$, $S_{\cup}^{\times}$, $S_{\cap}^{\times}$ and S_s be the subset pseudo interval semiring topological spaces associated with R.

Let $A = \begin{bmatrix} \{0.3, 6, 0.01, 0.2, 4, 5\} \\ \{0.01, 0.2, 0.04, 5, 7, 0.5\} \end{bmatrix}$

and $B = \begin{bmatrix} \{3, 6, 8, 1, 0.3, 0.9, 0.01\} \\ \{0.4, 4, 6, 0.2, 0.03, 0.009, 0.0005\} \end{bmatrix} \in T_o$;

$$A \cap B = \begin{bmatrix} \{0.3, 6, 0.01\} \\ \{0.2\} \end{bmatrix} \qquad \text{--- C}$$

$$A \cup B = \begin{bmatrix} \{0.3, 6, 0.01, 0.2, 4, 5, 8, 1, 3, 0.9\} \\ \{0.01, 0.2, 0.04, 5, 7, 0.5, 0.4, \\ 4, 6, 0.03, 0.009, 0.0005\} \end{bmatrix}$$

= D.

D, C $\in T_o$.

Let A, B $\in T_{\cup}^{+}$

A + B =

$$\begin{bmatrix} \begin{array}{l} \{3.3, 6.3, 8.3, 1.3, 0.6, 1.2, 0.31, 0, 3, 5, 7, 6.3, \\ 6.9, 6, 6.01, 3.01, 6.01, 8.01, 1.01, 0.91, 0.02, 3.2, \\ 5.9, 8, 6.2, 8.2, 0.5, 1.1, 0.21, 5.3, 2, 7, 1, 5, 4.3, \\ 4.9, 4.01, 5.5\} \end{array} \\ \begin{array}{l} \{0.41, 4.01, 7.03, 6.01, 7.009, 0.9, 4.5, 7.0005, 0.21, 0.6, \\ 5, 7.2, 6.2, 0.7, 0.5005, 0.209, 0.0045, 5.4, 0, 2, 5.2, 5.03, 4.2, \\ 0.04, 6.5, 0.019, 0.0105, 5.009, 0.24, 0.4, 0.23, \\ 0.509, 0.2005, 0.44, 4.04, 5.0005, 7.4, 0.53, \\ 6.04, 0.07, 0.049\} \end{array} \end{bmatrix} = E.$$

D, E $\in T_{\cup}^{+}$.

Let A, B $\in T_{\cap}^{+}$;

A + B = E and A $\cap$ B = C with E, C $\in T_{\cap}^{+}$.

Let A, B $\in T_{\cup}^{\times}$;

A $\cup$ B = D,

$$A \times B = \begin{bmatrix} \begin{matrix} \{0.9, 1.8, 2.4, 0.3, 0.09, 0.27, 2, 1.6, 0.003, \\ 0, 3, 6, 5.4, 8, 0.2, 0.06, 0.03, 0.06, 0.08, 4, \\ 0.18, 0.01, 0.003, 0.009, 3.6, 0.002, 0.0001, \\ 0.6, 1.2, 0.04, 5, 1.5, 4.5, 0.05\} \end{matrix} \\ \begin{matrix} \{0.004, 0.04, 0.06, 2, 0.002, 0.008, 1, 0.0003, \\ 0.063, 0.000005, 0.21, 0.0035, 0.2, 3, 0.1, \\ 2, 6, 0.08, 0.015, 0.0045, 0.00025, 0.8, 1.2, 0.006, \\ 0.0018, 0.15, 0.0001, 0.16, 0.24, 0.045, 0.0025, \\ 2.8, 4, 6, 1.4, 0.0012, 0.00036, 0.0002\} \end{matrix} \end{bmatrix}$$

$= F. D, F \in T_{\cup}^{\times}$.

Let $A, B \in T_{\cap}^{\times}$;

$A \cap B = C, A \times B = F \in T_{\cap}^{\times}$.

Let $A, B \in T_s$, $A \times B$ and $A + B \in T_s$ that is $F, E \in T_s$.

It is easily verified all the 6 super subset pseudo interval semiring topological spaces are distinct.

Let $A = \left\{ \begin{bmatrix} 0.3 \\ 6.2 \end{bmatrix}, \begin{bmatrix} 5.2 \\ 3.7 \end{bmatrix}, \begin{bmatrix} 1.43 \\ 0.72 \end{bmatrix}, \begin{bmatrix} 4.52 \\ 1.111 \end{bmatrix} \right\}$ and

$B = \left\{ \begin{bmatrix} 3 \\ 7 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \end{bmatrix} \right\} \in R$.

Let $A, B \in T_o$;

$A \cup B = \left\{ \begin{bmatrix} 0.3 \\ 6.2 \end{bmatrix}, \begin{bmatrix} 5.2 \\ 3.7 \end{bmatrix}, \begin{bmatrix} 1.43 \\ 0.72 \end{bmatrix}, \begin{bmatrix} 4.52 \\ 1.111 \end{bmatrix}, \right.$

$$\begin{bmatrix}3\\7\end{bmatrix},\begin{bmatrix}1\\0\end{bmatrix},\begin{bmatrix}2\\5\end{bmatrix},\begin{bmatrix}0\\5\end{bmatrix}\Bigg\}$$

$= C \in T_o$.

$A \cap B = \phi \in T_o$.

Let A, B $\in T_{\cup}^{+}$;

$$A + B = \left\{\begin{bmatrix}3.3\\4.2\end{bmatrix},\begin{bmatrix}1.3\\6.2\end{bmatrix},\begin{bmatrix}2.3\\2.2\end{bmatrix},\begin{bmatrix}0.3\\2.2\end{bmatrix},\begin{bmatrix}8.2\\1.7\end{bmatrix},\begin{bmatrix}6.2\\3.7\end{bmatrix},\right.$$

$$\begin{bmatrix}7.2\\8.7\end{bmatrix},\begin{bmatrix}5.2\\8.7\end{bmatrix},\begin{bmatrix}4.43\\7.72\end{bmatrix},\begin{bmatrix}2.43\\0.72\end{bmatrix},\begin{bmatrix}3.43\\5.72\end{bmatrix},\begin{bmatrix}1.43\\5.72\end{bmatrix},$$

$$\left.\begin{bmatrix}7.52\\8.111\end{bmatrix},\begin{bmatrix}5.52\\1.111\end{bmatrix},\begin{bmatrix}6.52\\6.111\end{bmatrix},\begin{bmatrix}4.52\\6.111\end{bmatrix}\right\} = D.$$

$A \cup B = C$, D, C $\in T_{\cup}^{+}$.

Consider A, B $\in T_{\cap}^{+}$;

A + B = D and $A \cap B = \phi \in T_{\cap}^{+}$.

Let A, B $\in T_{\cup}^{\times}$;

$A \cup B = C$,

$$A \times B = \left\{\begin{bmatrix}0.9\\3.4\end{bmatrix},\begin{bmatrix}6.6\\7.9\end{bmatrix},\begin{bmatrix}4.29\\5.04\end{bmatrix},\begin{bmatrix}4.56\\7.777\end{bmatrix},\begin{bmatrix}0.3\\0\end{bmatrix},\begin{bmatrix}5.2\\0\end{bmatrix},\right.$$

$$\begin{bmatrix}1.43\\0\end{bmatrix},\begin{bmatrix}4.52\\0\end{bmatrix},\begin{bmatrix}0.6\\4.0\end{bmatrix},\begin{bmatrix}1.4\\0.5\end{bmatrix},\begin{bmatrix}2.86\\3.60\end{bmatrix},\begin{bmatrix}0.04\\5.555\end{bmatrix},$$

$$\left.\begin{bmatrix}0\\4\end{bmatrix},\begin{bmatrix}0\\0.5\end{bmatrix},\begin{bmatrix}0\\3.6\end{bmatrix},\begin{bmatrix}0\\5.555\end{bmatrix}\right\} = E \in T_{\cup}^{\times}.$$

Now let A, B $\in T_{\cap}^{\times}$

$A \cap B = \phi$, $A \times B = E \in T_{\cap}^{\times}$.

Finally if A, B $\in T_s$.

We see A + B = D, $A \times B = E \in T_s$.

We see all the six semiring topological spaces are distinct.

Now all the six spaces associated with R is different from S.

Example 3.26: Let

$$S = \left\{\begin{bmatrix}a_1 & a_2 & a_3\\a_4 & a_5 & a_6\\a_7 & a_8 & a_9\end{bmatrix} \middle| a_i \in ([0, 12)), 1 \leq i \leq 9, +, \times\right\}$$

be the subset interval pseudo ring.

P = {Collection of all subsets of S}. P is a pseudo subset interval semiring of type I.

T_o, $T_{\cup}^{+}$, $T_{\cap}^{+}$, $T_{\cup}^{\times}$, $T_{\cap}^{\times}$ and T_s are the six subset pseudo interval semiring topological spaces.

We see all the spaces have infinite basis and each of the spaces are disconnected.

Example 3.27: Let S = {Collection of all subsets from the subset interval pseudo super matrix ring

$$P = \left\{ \left[\begin{array}{cc|ccc} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & \dots & \dots & \dots & a_{10} \\ \hline a_{11} & \dots & \dots & \dots & a_{15} \\ a_{16} & \dots & \dots & \dots & a_{20} \\ a_{21} & \dots & \dots & \dots & a_{25} \end{array} \right] \right| a_i \in ([0, 24)),$$

$$1 \leq i \leq 25, +, \times_n\}, \cup, \cap\}$$

be the pseudo subset semiring ring of type I.

Associated with S we get the 6 subset interval pseudo semiring topological spaces.

Example 3.28: Let W = {Collection of all subsets from the subset interval pseudo ring

$$P = \left\{ \left[\begin{array}{cccc} a_1 & a_2 & \dots & a_9 \\ a_{10} & a_{11} & \dots & a_{18} \\ a_{19} & a_{20} & \dots & a_{27} \\ a_{28} & a_{29} & \dots & a_{36} \end{array} \right] \right| a_i \in ([0, 15)),$$

$$1 \leq i \leq 36, +, \times_n\}, \cup, \cap\}$$

be the subset interval pseudo semiring ring.

We have the associated semiring topological spaces.

Example 3.29: Let M = {Collection of all subsets from the subset interval pseudo ring

$$P = \left\{ \left[\begin{array}{c} a_1 \\ a_2 \\ \vdots \\ a_9 \end{array} \right] \right| a_i \in [0, 19), 1 \leq i \leq 9, +, \times_n\}, \cup, \cap\}$$

be the subset interval pseudo semiring ring.

Let T_o, $T_{\cup}^{+}$, $T_{\cap}^{+}$, $T_{\cup}^{\times}$, $T_{\cap}^{\times}$ and T_s are the pseudo subset interval semiring topological spaces.

$T_{\cup}^{\times}$, $T_{\cap}^{\times}$ and T_s have infinite number of zero divisors.

Next we consider type II subset interval semiring and the semiring topological spaces associated with them.

DEFINITION 3.3: *Let S = {Collection of all subsets from the subset interval semiring.*

P = {[0,n), min, max}} be the subset interval semiring under $\cup$ and $\cap$ of type II.

$T_o = \{S' = S \cup \{\phi\}, \cup, \cap\}$

$T_{\cup}^{min} = \{S, \cup, min\}$, $T_{\cup}^{max} = \{S, \cup, max\}$,

$T_{\cap}^{min} = \{S', \cup, min\}$, $T_{\cap}^{max} = \{S', \cap, max\}$ *and*

$T_s = \{S, min, max\}$ *be the six subset interval semiring topological spaces of type II associated with S.*

We will illustrate this situation by some examples.

Example 3.30: Let W = {Collection of all subsets of the semiring S = {[0, 12), min, max}, $\cup$, $\cap$}} be the type II interval subset semiring.

Let T_o, $T_{\cup}^{min}$, $T_{\cap}^{min}$, $T_{\cup}^{max}$, $T_{\cap}^{max}$ and T_s be the six subset interval special semiring topological spaces of type II associated with W.

Let A = {0.8, 4, 8, 7.1,5, 5.2} and

$B = \{1, 0.1, 0.02, 0.06, 5, 4, 9, 0.008, 0.7\} \in T_o$.

$A \cap B = \{5, 4\}$ --- C

$A \cup B = \{0.8, 4,8, 7.1, 5, 0.006,5.2, 1, 0.1, 0.02, 9, 0.008, 0.7\}$ --- D

Clearly C, D $\in T_o$.

Let A, B $\in T_{\cup}^{\min}$,

$A \cup B = D$

and min $\{A, B\} = \{0.8, 0.1, 0.02, 0.006, 0.008, 0.7, 1, 4, 5, 7.1\} = E$.

Clearly E, D $\in T_{\cup}^{\min}$.

$T_{\cup}^{\min}$ and T_o are different as subset interval semiring semiring topological space of type II.

Let A, B $\in T_{\cap}^{\min}$;

min $\{A, B\} = E$ and
$A \cap B = C$; E, C $\in T_{\cap}^{\min}$.

$T_{\cap}^{\min}$ is different from T_o and $T_{\cup}^{\min}$.

Let A, B $\in T_{\cup}^{\max}$

max $\{A, B\} = \{1, 5, 4, 9, 0.8, 8, 7.1, 5.2\} = F$

$A \cup B = D$ and F, D $\in T_{\cup}^{\max}$.

$T_{\cup}^{\max}$ is different from T_o, $T_{\cup}^{\min}$ and $T_{\cap}^{\min}$.

Consider A, B $\in T_{\cap}^{\max}$; A $\cap$ B = C
and max {A, B} = F, F, C $\in T_{\cap}^{\max}$.

$T_{\cap}^{\max}$ is different from T_o, $T_{\cup}^{\min}$, $T_{\cap}^{\min}$, $T_{\cup}^{\max}$ and $T_{\cap}^{\max}$.

Let A, B $\in T_s$.

min {A, B} = E and max {A, B} = F; E, F $\in T_s$.

T_s is different from $T_{\cup}^{\min}$, $T_{\cap}^{\min}$, $T_{\cup}^{\max}$ and $T_{\cap}^{\max}$.

Thus all the six subset interval semiring semiring topological spaces of type I are different.

Example 3.31: Let S = {Collection of all subsets from the subset interval matrix semiring

$$R = \left\{\left[\begin{matrix} a_1 \\ a_2 \end{matrix}\right] \middle|\ a_1, a_2 \in [0, 7), \min, \max\right\}, \cup, \cap\}$$

be the subset interval semiring semiring of type II.

T_o, $T_{\cup}^{\min}$, $T_{\cap}^{\min}$, $T_{\cup}^{\max}$, $T_{\cap}^{\max}$ and T_s be the associated subset interval semiring semiring topological spaces of type II.

All these spaces are distinct.

All of them have an infinite basis.

However these spaces have subspaces of finite cardinality.

Let P_1 = {Collection of all subsets of the subset interval subsemiring

$$B_1 = \left\{\left[\begin{matrix} a_1 \\ 0 \end{matrix}\right] \middle|\ a_1 \in Z_7, \min, \max\right\}, \cup, \cap\} \subseteq S$$

is a subset interval subsemiring of type II.

All the six topological spaces associated with B_1 is finite and has a finite basis.

Example 3.32: Let S = {Collection of all subsets from the subset interval semiring

$$R = \left\{ \begin{bmatrix} a_1 & a_2 & \dots & a_9 \\ a_{10} & a_{11} & \dots & a_{18} \\ a_{19} & a_{20} & \dots & a_{27} \\ a_{28} & a_{29} & \dots & a_{36} \end{bmatrix} \middle| \; a_i \in [0, 16), 1 \leq i \leq 36, \right.$$

$$\text{min, max}\}, \cup, \cap\}$$

be the subset interval semiring semiring of type II.

All the 6 subset interval semiring topological spaces of type II associated with S have infinite basis.

All of them are disconnected.

However these have topological subspaces of finite cardinality and hence has finite basis.

Example 3.33: Let B = {Collection of all subsets from the subset interval semiring

$$P = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \end{bmatrix} \middle| \; a_i \in [0, 15), 1 \leq i \leq 15, \right.$$

$$\text{min, max}\}, \cup, \cap\}$$

be the special subset interval semiring semiring of type II.

We have pairs of subspaces in $T_{\cup}^{\min}$ and $T_{\cap}^{\min}$ such that min {A, B} = {0} and A ∩ B = {0} in case of A, B ∈ $T_{A \cap B}^{\min}$.

Let A = {Collection of all subsets from

$$X = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{bmatrix} \middle| \; a_i \in [0, 15), 1 \le i \le 3, \min, \max\}, \cup, \cap\} \right.$$

be the subset interval semiring subsemiring of type II.

Let B = {Collection of all subsets of

$$Y = \left\{ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ a_1 & a_2 & a_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \middle| \; a_i \in [0, 15), 1 \le i \le 3, \min, \max\}, \cup, \cap\} \right.$$

be the subset interval subsemiring of type II.

Clearly min {A, B} = {min {a, b} | a ∈ A and b ∈ B}

$$= \left\{ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{bmatrix} \right\}.$$

If a, b ∈ $T_{\cap}^{\min}$ then

$$A \cap B = \{a \cap b \mid a, b \in T_{\cap}^{\min} = \left\{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{bmatrix}\right\}.$$

We have several such pairs of subspaces of $T_{\cap}^{\min}$ ($T_{\cup}^{\min}$).

Study of this is put forth in the following theorem.

THEOREM 3.6: *Let M = {Collection of all subsets from the subset interval semiring S ={m × n matrices with entries from [0, s); min, max} ∪, ∩} be the subset interval semiring of type II.*

(i) M has associated with it six distinct subset interval semiring topological spaces of type II viz, T_o, $T_{\cup}^{min}$, $T_{\cap}^{min}$, $T_{\cup}^{max}$, $T_{\cap}^{max}$ *and* T_s.

(ii) $T_{\cup}^{min}$, $T_{\cap}^{min}$ *and* T_s *has pairs of subspaces A, B such that min {A, B} = {min {a, b} | a ∈ A and b ∈ B, A, B subspace of* $T_{\cap}^{min}$ *(or* $T_{\cup}^{max}$ *or* T_s*)} = {m × n zero matrix}.*

Proof is direct and hence left as an exercise to the reader.

Note thus we have the concept of annihilating subset interval topological subspaces of type II.

For instance for a given subspace A we can have several subspaces B_i, $1 \leq i \leq n$ such that $\min\{A, B_i\} = \{(0)\}$, $1 \leq i \leq n$.

We will just illustrate this situation by an example.

Example 3.34: Let S ={Collection of all subsets from the subset interval semiring P ={$(a_1, a_2, a_3, a_4) \mid a_i \in [0, 20)$, $1 \leq i \leq 4$, min, max}, ∪, ∩} be the subset interval semiring of type II.

Let T_o, $T_{\cup}^{\min}$, $T_{\cap}^{\min}$, $T_{\cup}^{\max}$, $T_{\cap}^{\max}$ and T_s be subset interval semiring topological spaces of type II.

If A ={(a_1, 0, 0, 0) | $a_1 \in$ [0, 20)} and X_A = {Collection of all subsets of the semiring A} $\subseteq T_{\cup}^{\min}$ or $T_{\cap}^{\min}$ or T_s then X_B = {Collection of all subsets of {(0, b, 0, 0) b $\in$ [0, 20), min, max}, $\cup$, $\cap$} $\subseteq T_{\cup}^{\min}$ (or $T_{\cap}^{\min}$ or T_s) is such that

$$\min \{X_A, X_B\} = \{(0, 0, 0, 0\}.$$

Likewise X_C = {Collection of all subsets from C = {(0, 0, c, 0) | c $\in$ [0, 20), min, max}, $\cup$, $\cap$} be the subspace of $T_{\cup}^{\min}$ (or , $T_{\cap}^{\min}$ or T_s) then min {X_C, X_A} = (0) and

$$\min \{X_C, X_B\} = \{(0, 0, 0, 0)\}.$$

X_D = {Collection of all subsets from D = {(0, 0, 0, d) | d $\in$ [0,20), min, max}, $\cup$, $\cap$} $\subseteq T_{\cup}^{\min}$ (or , $T_{\cap}^{\min}$ or T_s) be the subset interval semiring topological subspace of type II.

$$\min \{X_D, X_A\} = \{(0, 0, 0, 0)\},$$
$$\min \{X_D, X_B\} = \{(0, 0, 0, 0)\} \text{ and}$$
$$\min \{X_D, X_C\} = \{(0, 0, 0, 0)\}.$$

X_E = {Collection of all subsets from the subsemiring E = {(0, x, y, 0) | x, y $\in$ [0, 20), min, max}, $\cup$, $\cap$} $\subseteq T_{\cup}^{\min}$ (or , $T_{\cap}^{\min}$ or T_s) be the subset interval semiring subtopological subspace of type II.

Clearly $\min \{X_A, X_E\} = \{(0, 0, 0, 0)\}$.

X_F = {Collection of all subsets from the subsemiring F = {(0, x, 0, y) | x, y $\in$ [0, 20), min, max}, $\cup$, $\cap$} $\subseteq T_{\cup}^{\min}$ (or , $T_{\cap}^{\min}$ or T_s) be the subset interval semiring subtopological subspace of type II.

Clearly $\min \{X_A, X_F\} = \{(0, 0, 0, 0)\}$.

Now take X_G = {Collection of all subsets from the subset interval subsemiring G = {(0, 0, x, y) | x, y $\in$ [0, 20), min, max}, $\cup$, $\cap$} $\subseteq$ $T_{\cup}^{\min}$ (or , $T_{\cap}^{\min}$ or T_s) be the subset interval semiring subtopological subspace of type II.

We have min {X_A, X_G} = {(0, 0, 0, 0)}.

Finally X_H = {Collection of all subsets from the subset interval subsemiring H = {(0, x, y, z) | x, y, z $\in$ [0, 20), min, max}, $\cup$, $\cap$} $\subseteq$ $T_{\cup}^{\min}$ (or , $T_{\cap}^{\min}$ or T_s) be the subset interval semiring subtopological subspace of type II.

We see min {X_A, X_H} = {(0, 0, 0, 0)}.

Thus for the subspace X_A we have X_B, X_C, X_D, X_E, X_F and X_G atleast 6 subspaces to be annihilating subspaces of X_A.

Infact we have more subspaces which annihilate X_A or a zero divisor subspace of finite order also.

Take P_1 = {Collection of all subsets of the interval subset semiring; L_1 = {(0, a, 0, 0) | a $\in$ Z_{20}, min, max}, $\cup$, $\cap$} $\subseteq$ $T_{\cup}^{\min}$ ($T_{\cap}^{\min}$ or T_s) be the subset interval semiring subtopological subspace of type II.

We see min {X_A, P_1} = {(0, 0, 0, 0)}; we have several such finite cardinality subspaces also which have zero divisors.

Finally we can study the entire chapter three by replacing the interval [0, n) by C([0, n)) and all results hold good.

Similarly we can study replacing [0, n) by the neutrosophic interval ⟨[0, n) $\cup$ I⟩ and the results in the entire chapter three are true.

Finally we can make a study by replacing [0, n) by C(⟨[0, n) $\cup$ I⟩) and all the results hold good.

All this is left as an exercise to the reader. We supply a few examples of them.

Example 3.35: Let M ={Collection of all subsets from the subset interval complex modulo integer semiring

$$P = \left\{\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \middle|\ a_i \in C([0, 10)), 1 \leq i \leq 4, \min, \max\}, \cup, \cap\right\}$$

be the subset interval finite complex modulo integer semiring of type II.

T_o, $T_{\cup}^{\min}$, $T_{\cap}^{\min}$, $T_{\cup}^{\max}$, $T_{\cap}^{\max}$ and T_s be the subset interval complex modulo integer semiring topological space of type II.

All properties can be arrived with no difficulty.

Example 3.36: Let M ={Collection of all subsets from the subset interval neutrosophic semiring

$$P = \left\{\begin{bmatrix} a_1 & a_{10} \\ a_2 & a_{11} \\ \vdots & \vdots \\ a_9 & a_{18} \end{bmatrix} \middle|\ a_i \in C(\langle[0, 12) \cup I\rangle),\right.$$

$$\left. 1 \leq i \leq 18, \min, \max\}, \cup, \cap\right\}$$

be the subset interval neutrosophic semiring of type II. Then T_o, T_s, $T_{\cup}^{\min}$, $T_{\cap}^{\min}$, $T_{\cup}^{\max}$ and $T_{\cap}^{\max}$ are the six subset interval neutrosophic semiring topological space of type II.

Example 3.37: Let M ={Collection of all subsets from the finite complex modulo integer neutrosophic subset interval semiring

$$L = \left\{ \left. \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_9 \end{bmatrix} \right| a_i \in C(\langle [0, 12) \cup I \rangle), 1 \leq i \leq 9, \min, \max\}, \cup, \cap \right\}$$

be the subset interval finite complex modulo integer neutrosophic semiring of type II. T_o, $T_{\cup}^{\min}$, $T_{\cap}^{\min}$, $T_{\cup}^{\max}$, $T_{\cap}^{\max}$ and T_s are the six subset interval finite complex modulo integer neutrosophic semiring topological spaces of type II associated with S.

All the properties can be derived in case of these spaces also.

Now we proceed onto define type II pseudo subset interval semirings and the pseudo topological space associated with them.

DEFINITION 3.4: *Let S = {Collection of all subsets from the pseudo interval subset semiring P= {[0, n), min, ×}, ∪, ∩} be the pseudo subset interval semiring of type II. Associated with S we have the six pseudo subset interval semiring topological spaces of type II viz.,* T_o, $T_{\cup}^{\min}$, $T_{\cap}^{\min}$, $T_{\cup}^{\max}$, $T_{\cap}^{\max}$ *and* T_s.

We will first illustrate this situation by some examples.

Example 3.38: Let M ={Collection of all subsets from the pseudo subset interval semiring R = $\{(a_1, a_2, a_3) \mid a_i \in [0, 12), 1 \leq i \leq 3, \min, \times\}, \cup, \cap\}$ be the subset interval pseudo semiring of type II. T_o, $T_{\cup}^{\min}$, $T_{\cap}^{\min}$, $T_{\cup}^{\max}$, $T_{\cap}^{\max}$ and T_s are the pseudo subset interval semiring topological spaces of type II.

We see all the six spaces are distinct.

For let A = {(0, 3, 1.4), (5, 2, 1), (7, 21, 4), (6.1, 2.7, 0)} and

B = {(0.3, 1, 2), (0.6, 0.1, 0.2), (0.004, 0.5, 0.7), (0.009, 0.5,0.7)} $\in T_o$.

A $\cup$ B = {(0,3, 1.4), (5, 2, 1), (7, 2.1, 4), (6.1,2.7,0), (0.3, 1, 2), (0.6, 0.1, 0.2), (0.004, 0.008, 0), (0.009, 0.5, 0.7)} = C

A $\cap$ B = {ϕ} --- D

C, D $\in T_o$.

Let A, B $\in T_{\cup}^{\min}$;

A $\cup$ B = C and
min {A, B} = {(0, 1, 1.4), (0, 0.1, 0.2), (0, 0.008, 0), (0.0009, 0.5,0.7), (0.3, 1,1), (0.6, 0.1, 0.2), (0.004, 0.008, 0), (0.0009, 0.5, 0), (0.3, 1, 1.4), (0.004, 0.008, 0), (0.3,1,0), (0.6, 0.1,0)} = E.

C, E $\in T_{\cup}^{\min}$.

Clearly T_o is different from $T_{\cup}^{\min}$.

Let A, B $\in T_{\cap}^{\min}$.

A $\cap$ B = {ϕ} and min {A, B} = E, D and E $\in T_{\cap}^{\min}$.

$T_{\cap}^{\min}$ is different from $T_{\cup}^{\min}$ and $T_{\cap}^{\min}$.

Consider A, B $\in T_{\cup}^{\times}$;

A $\cup$ B = C and A $\times$ B = {(0, 3, 2.8), (1.5, 2, 2), (1.83, 2.7, 0), (2.1, 2.1, 8), (0, 0.3, 0.28), (3, 0.2, 0.2), (4.2, 0.21, 0.8), (3.66, 0.27, 0), (0, 0.024, 0), (0.02, 0.016, 0), (0.028, 0.0168, 0), (0.0244, 0.0216, 0), (0, 1.5, 0.98),(0.0045, 1, 0.7), (0.0063, 1.05, 2.8), (0.00549, 1.35, 0)} = F.

C, F $\in T_{\cup}^{\times}$ and $T_{\cap}^{\times}$ is different from T_o, $T_{\cup}^{\min}$ and $T_{\cap}^{\min}$.

Consider A, B $\in T_{\cup}^{\times}$;

$A \cap B = \{\phi\} = D$ and $A \times B = F$; D, F $\in T_{\cap}^{\times}$ and $T_{\cap}^{\times}$ is different from T_o, $T_{\cup}^{\times}$, $T_{\cup}^{\min}$ and $T_{\cap}^{\min}$.

Finally let A, B $\in T_s$.

$A \times B = F$ and $\min \{A, B\} = E$ and F, E $\in T_s$.

T_s is different from all the other five subset interval pseudo semiring topological spaces of type II.

Example 3.39: Let S = {Collection of all subsets from the pseudo subset interval semiring P = $\{(a_1, a_2, \ldots, a_6) \mid a_i \in C([0, 24)), 1 \leq i \leq 6, \min, \times\}, \cup, \cap\}$ be the subset interval pseudo finite complex modulo integer semiring of type II.

All the six pseudo subset interval semiring topological spaces of type II associated with S are distinct.

Infact the spaces $T_{\cup}^{\times}$, $T_{\cap}^{\times}$, $T_{\cup}^{\min}$, $T_{\cap}^{\min}$ and T_s have pairs of zero subspaces.

Example 3.40: Let

$$B = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_9 \end{bmatrix} \middle| \; a_i \in C(\langle [0, 20) \cup I \rangle), 1 \leq i \leq 9, \min, \times \right\}$$

be the subset interval pseudo semiring of neutrosophic column matrices.

S = {Collection of all subsets from the subset interval pseudo neutrosophic semiring B}, ∪, ∩} is the subset interval neutrosophic subset pseudo semiring of type II.

Associated with S we have 6 distinct pseudo subset interval semiring topological spaces. The pseudo semiring topological spaces T_o, $T_{\cup}^{\min}$, $T_{\cap}^{\min}$, $T_{\cup}^{\times}$ and $T_{\cap}^{\times}$ contain pair of pseudo subspaces A, B which are such that A × B = {(0)} {and min{A, B} = {(0)}}.

Example 3.41: Let S ={Collection of all subsets of the pseudo subset interval finite complex modulo integer semiring

$$T = \left\{ \left. \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \right| a_i \in C([0, 12)), 1 \leq i \leq 9, \min, \times \right\}, \cup, \cap \}$$

be the pseudo subset interval semiring of type II. T_o, $T_{\cup}^{\min}$, $T_{\cap}^{\min}$, $T_{\cup}^{\times}$, $T_{\cap}^{\times}$ and T_s are the associated six pseudo subset interval semiring complex finite modulo integer topological spaces of type II.

Infact $T_{\cup}^{\min}$, $T_{\cap}^{\min}$, $T_{\cup}^{\times}$, $T_{\cap}^{\times}$ and T_s contain pseudo subspace pairs such A × B = {(0)} and min {A, B} = {(0)}.

Example 3.42: Let S ={Collection of all subsets from the subset interval pseudo semiring

$$P = \left\{ \left. \begin{bmatrix} a_1 & a_2 & \dots & a_{10} \\ a_{11} & a_{12} & \dots & a_{20} \\ a_{21} & a_{22} & \dots & a_{30} \end{bmatrix} \right| a_i \in C(\langle [0, 11) \cup I \rangle), \right.$$

$$1 \leq i \leq 30, \min, \times_n \}, \cup, \cap \}$$

be the pseudo subset interval semiring of type II.

All the six pseudo subset interval semiring topological spaces are of infinite cardinality and $T_{\cup}^{\min}$, $T_{\cap}^{\min}$, $T_{\cup}^{\times}$, $T_{\cap}^{\times}$ and T_s contain pairs of subtopological spaces A, B and A × B = {(0)}.

Example 3.43: Let

$$B = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{28} & a_{29} & a_{30} \end{bmatrix} \middle| \; a_i \in C(\langle [0, 29) \cup I \rangle), 1 \leq i \leq 30, \min, \times_n \right\}$$

be the subset interval pseudo finite complex modulo integer neutrosophic semiring.

S = {Collection of all subsets from B, ∪, ∩} be the subset interval finite complex modulo integer neutrosophic pseudo semiring of type II. All the associated six pseudo semiring topological spaces of type II are distinct.

All properties in case of pseudo topological semiring spaces of type II can be studied with out any difficulty.

We now proceed onto propose a few problems for this chapter.

Problems:

1. Find the special properties enjoyed by the subset super interval semigroups.

2. Show super subset interval semigroups under product can have zero divisors.

3. Prove all subset super interval semigroups are of infinite cardinality.

4. Show super subset interval semigroups can have subsemigroups of finite order.

5. Obtain some special features enjoyed by super subset interval semigroup topological spaces.

6. Let W = {Collection of all subsets from the subset interval semigroup, P = {$(a_1, a_2, \ldots, a_9)$ | $a_i \in [0, 23)$, $1 \leq i \leq 9$}, max} be the super subset interval semigroup under max.

   (i) Find $T_{\cup}^{\max}$, $T_{\cap}^{\max}$ and T_o, the super subset interval semigroup topological spaces associated with W.

   (ii) Prove all the three spaces have infinite basis.

   (iii) Prove all the three spaces have subspaces of finite cardinality.

   (iv) Obtain any other special feature associated with these spaces.

7. Let S = {Collection of all subsets from the subset complex finite modulo integer matrix semigroup

$$P = \left\{ \left[\begin{array}{cc} a_1 & a_{10} \\ a_2 & a_{11} \\ \vdots & \vdots \\ a_9 & a_{18} \end{array} \right] \middle| \; a_i \in C([0, 21)), 1 \leq i \leq 18, \max \right\},$$

   max} be super subset interval semigroup.

   Study questions (i) to (iv) of problem 6 for this S.

8. Let S = {Collection of all subsets from the subset interval neutrosophic semigroup

$$P = \left\{ \left[\begin{array}{cc|cc} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ \hline a_{13} & a_{14} & a_{15} & a_{16} \\ a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{array} \right] \right| a_i \in \langle [0, 20) \cup I \rangle,$$

$1 \leq i \leq 24$, max}, max} be the subset super interval semigroup.

Study questions (i) to (iv) of problem 6 for this S.

9. Let S = {Collection of all subsets from the subset interval finite complex modulo integer neutrosophic semigroup;

$$B = \left\{ \left(\begin{array}{c|ccc|cc|ccc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 \\ a_{10} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{18} \end{array} \right) \right|$$

$a_i \in \langle [0, 12) \cup I \rangle$, $1 \leq i \leq 18$, max}, max} be the super subset interval finite complex modulo integer neutrosophic semigroup.

Study questions (i) to (iv) of problem 6 for this S.

10. Let $B = \left\{ \left[\begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{array} \right] \right| a_i \in [0, 11), 1 \leq i \leq 16,$

min} be the subset interval matrix semigroup.
Let W = {Collection of all subsets from the subset interval semigroup B, min} be the super subset interval semigroup.

Study questions (i) to (iv) of problem 6 for this B.

11. Let M = $\left\{\begin{pmatrix} a_1 & a_2 & \dots & a_9 \\ a_{10} & a_{11} & \dots & a_{18} \end{pmatrix} \middle| \; a_i \in C([0, 124)),\right.$

    $1 \leq i \leq 18$, min} be the subset interval semigroup of complex finite modulo integers.
    R = {Collection of all subsets of M, min} be the super subset interval semigroup.

    Study questions (i) to (iv) of problem 6 for this M.

12. Let M = {Collection of all subsets from the subset interval complex finite modulo integer neutrosophic

    semiring S = $\left\{\begin{bmatrix} a_1 \\ \hline a_2 \\ a_3 \\ a_4 \\ \hline a_5 \\ a_6 \\ \hline a_7 \\ a_8 \end{bmatrix} \middle| \; a_i \in C(\langle [0, 5) \cup I \rangle), 1 \leq i \leq 8,\right.$

    min}, min} be the super subset interval super matrix.

    Study questions (i) to (iv) of problem 6 for this M.

13. Let S = {Collection of all subsets from the subset interval semigroup
    P = $\{(a_1, a_2, \dots, a_{10}) \mid a_i \in [0, 14); 1 \leq i \leq 10, \times\}, \times\}$ be the subset interval super semigroup.

(i) Prove associated with S the topological spaces $T_{\cup}^{\times}$ and $T_{\cup}^{\times}$ have pairs of subspaces A, B such that $A \times B = \{(0)\}$.

(ii) Study questions (i) to (iv) of problem 6 for this S.

14. Let S = {Collection of all subsets from the subset

interval semigroup M = $\left\{\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \\ a_{11} & a_{12} \\ a_{13} & a_{14} \end{bmatrix} \middle| a_i \in C([0, 43)),\right.$

$1 \leq i \leq 14, \times_n\}, \times_n\}$ be the super subset interval finite complex modulo integer semigroup.

Study questions (i) and (ii) of problem 13 for this S.

15. Let S = {Collection of all subsets from the subset

interval semigroup M = $\left\{\begin{bmatrix} a_1 & a_2 & \cdots & a_9 \\ a_{10} & a_{11} & \cdots & a_{18} \\ a_{19} & a_{20} & \cdots & a_{27} \\ a_{28} & a_{29} & \cdots & a_{36} \\ a_{37} & a_{38} & \cdots & a_{45} \end{bmatrix} \middle| a_i \in\right.$

$C(\langle[0, 10) \cup I\rangle, 1 \leq i \leq 45, \times_n\}, \times_n\}$ be the super subset interval semigroup of finite complex modulo integer neutrosophic matrices.

Study questions (i) and (ii) of problem 13 for this S.

16. Let S = {Collection of all subsets from the subset interval semigroup P = $\left\{\begin{bmatrix} a_1 & a_2 & \ldots & a_{15} \\ a_{16} & a_{17} & \ldots & a_{30} \\ a_{31} & a_{32} & \ldots & a_{45} \end{bmatrix} \middle| a_i \in\right.$ C(⟨[0, 40) ∪ I⟩), $1 \leq i \leq 45$, $\times_n$}, $\times_n$} be the super subset interval semigroup of complex neutrosophic matrices.

    Study questions (i) and (ii) of problem 13 for this S.

17. Obtain some special features enjoyed by the super subset interval semigroup topological spaces T_o, $T_{\cup}^{\times}$ and $T_{\cap}^{\times}$.

    (i) Are these spaces disconnected?
    (ii) Do they form metric spaces?
    (That is can any type of metric be defined on T_o, $T_{\cup}^{\times}$ and $T_{\cap}^{\times}$).
    (iii) Are these spaces compact?
    (iv) Can these spaces be Hausdorff?
    (v) Are there spaces locally compact?
    (vi) Mention any other feature enjoyed by these spaces.

18. Let S = {Collection of all subsets from the interval subset semiring P = {$(a_1, \ldots, a_7)$ | $a_i \in [0, 25)$, $1 \leq i \leq 7$, min, max}, ∪, ∩} be the super subset interval semiring. T_o, $T_{\cup}^{\min}$, $T_{\cap}^{\min}$, $T_{\cup}^{\max}$, $T_{\cap}^{\max}$ and T_s be the six super subset interval semiring topological spaces associated with S.

    (i) Find the special features enjoyed by these spaces.
    (ii) Study questions (i) to (vi) of problem 17 for these spaces.
    (iii) Can these spaces have subspaces with finite basis?
    (iv) Prove all the six spaces have infinite basis.
    (v) Show all these 6 spaces are distinct.

19. Let S = {Collection of all subsets from the subset

$$\text{interval semiring } B = \left\{ \left[\begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ a_5 & \ldots & \ldots & a_8 \\ a_9 & \ldots & \ldots & a_{12} \\ a_{13} & \ldots & \ldots & a_{16} \\ a_{17} & \ldots & \ldots & a_{20} \end{array}\right] \right| a_i \in$$

C([0, 23), $1 \leq i \leq 20$, min, max}, ∪, ∩} be the super subset interval semiring.

Study questions (i) and (v) of problem 17 for this S.

20. Let S = {Collection of all subsets from the subset

$$\text{interval semiring } B = \left\{ \left[\begin{array}{c} a_1 \\ \hline a_2 \\ a_3 \\ a_4 \\ \hline a_5 \\ a_6 \\ \hline a_7 \\ a_8 \\ a_9 \\ \hline a_{10} \end{array}\right] \right| a_i \in C(\langle[0, 43) \cup I\rangle),$$

$1 \leq i \leq 10$, min, max}, ∪, ∩} be the super subset interval semiring of finite complex modulo neutrosophic numbers.

Study questions (i) and (v) of problem 17 for this S.

21. Let S = {Collection of all subsets from the subset

$$\text{interval semiring } B = \left\{ \left[\begin{array}{cc|ccc} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & \ldots & \ldots & \ldots & a_{10} \\ a_{11} & \ldots & \ldots & \ldots & a_{15} \\ \hline a_{16} & \ldots & \ldots & \ldots & a_{20} \\ a_{21} & \ldots & \ldots & \ldots & a_{25} \\ a_{26} & \ldots & \ldots & \ldots & a_{30} \\ \hline a_{31} & \ldots & \ldots & \ldots & a_{35} \\ a_{36} & \ldots & \ldots & \ldots & a_{40} \end{array} \right] \right| a_i \in$$

C([0, 12), 1 ≤ i ≤ 40, min, max}, ∪, ∩} be the super subset interval finite complex modulo integer super matrix semiring.

Study questions (i) and (v) of problem 17 for this S.

22. Let S = {Collection of all subsets from the pseudo subset interval semiring B ={$(a_1 \mid a_2\ a_3\ a_4 \mid a_5)$ | $a_i \in$ [0, 43), 1 ≤ i ≤ 5, min, ×}, ∪, ∩} be the super subset interval semiring.
T_o, $T_{\cup}^{\times}$, $T_{\cap}^{\times}$, $T_{\cup}^{\min}$, $T_{\cap}^{\min}$ and T_s be the six pseudo semiring topological spaces associated with S.

(i) Prove all the six spaces are distinct.
(ii) Prove all these six spaces have infinite basis.
(iii) Prove all these spaces have subspace of finite cardinality.
(iv) Study questions (i) to (vi) of problem 17 for these six spaces.
(v) How does a pseudo semiring topological space different from the semiring topological space?
(vi) Obtain any other special feature enjoyed by these super special interval pseudo semiring topological spaces.

23. Let S = {Collection of all subsets from the pseudo subset interval semiring of finite complex modulo

$$\text{integer matrix } B = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} \\ a_{19} & a_{20} & a_{21} \\ a_{22} & a_{23} & a_{24} \\ a_{25} & a_{26} & a_{27} \end{bmatrix} \middle| \; a_i \in C([0, 27)), \right.$$

$1 \leq i \leq 27$, min, $\times_n$}, $\cup$, $\cap$} be the super subset interval matrix semiring.

Study questions (i) and (vi) of problem 22 for this S.

24. Let S = {Collection of all subsets of the subset interval neutrosophic matrix pseudo semiring

$$B = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & \dots & \dots & \dots & a_{10} \\ a_{11} & \dots & \dots & \dots & a_{15} \\ a_{16} & \dots & \dots & \dots & a_{20} \\ a_{21} & \dots & \dots & \dots & a_{25} \\ a_{26} & \dots & \dots & \dots & a_{30} \\ a_{31} & \dots & \dots & \dots & a_{35} \end{bmatrix} \middle| \; a_i \in C(\langle [0, 10) \cup I \rangle), \right.$$

$1 \leq i \leq 35$, min, $\times_n$}, $\cup$, $\cap$} be the super subset interval pseudo neutrosophic matrix semiring.

Study questions (i) and (vi) of problem 22 for this S.

25. Let B = {Collection of all subsets from the pseudo finite complex modulo integer neutrosophic subset interval

semiring T= $\left\{\left.\begin{pmatrix} a_1 & a_2 & \dots & a_{13} \\ a_{14} & a_{15} & \dots & a_{26} \\ a_{27} & a_{28} & \dots & a_{39} \\ a_{40} & a_{41} & \dots & a_{52} \end{pmatrix}\right| a_i \in C(\langle[0, 15) \cup I\rangle),\right.$

$1 \leq i \leq 52, +, \times_n\}, \cup, \cap\}$ be the super subset interval finite complex modulo integer neutrosophic semiring.

Study questions (i) and (vi) of problem 22 for this B.

26. If the interval [0, n) is replaced by SS([0, n)) what difference does that make?

27. Study questions 24, 25 and 26 by replacing the intervals by SS([0, n)).

28. When will the space be the same even if [0, n) is replaced by SS([0, n))?

29. Let M = {Collection of all subsets from the pseudo

subset interval semiring P = $\left\{\left.\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix}\right| a_i \in [0,14),\right.$

$1 \leq i \leq 5$, min, max}, $\cup$, $\cap$} be the subset interval semiring and

$$N = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} \middle| \ a_i \in SS([0,14), 1 \le i \le 5, \min, \max\} \right.$$

be the super special subset semiring.

(i) Compare M and N.
(ii) Is $M \cong N$?
(iii) Is $M \subseteq N$?
(iv) Is $N \subseteq M$?
(v) Compare the corresponding six semiring topological spaces.

30. Let $M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \middle| \ a_i \in C(\langle [0, 17) \cup I \rangle), 1 \le i \le 9, \right.$

min, $\times_n\}$ be the super special subset interval pseudo semiring and N = {Collection of all subsets from the

pseudo subset interval semiring $P = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \middle| \ a_i \right.$

$\in C(\langle [0, 17) \cup I \rangle)$, $1 \le i \le 9$, min, $\times_n\}$.

Study questions (i) and (v) of problem 29 for this M and N.

Chapter Four

STRONG SUPER SPECIAL SUBSET INTERVAL SUBSET TOPOLOGICAL SPACES

In this chapter we for the first time define a new class of strong super special subset interval topological spaces shortly represented as SSSS-interval topological spaces using semigroups or pseudo groups or semirings or pseudo rings.

Throughout this chapter S([0, n)) denotes the collection of all subsets which include also natural class of intervals from the interval [0, n) and is defined as the strong super special subset interval set.

On S([0, n)) we can define min operation or max operation or × operation or + operation or so on. We will first illustrate this by some examples.

Example 4.1: Let S([0, 5)) be the collection of all subsets

P = {(0.7, 1), (1, 0.7), (0.3, 0.2], [0.752, 0.3)} and
Q = {4, 0.221, 4.301, 2.221, 1.06, [0, 2], [0, 0.7)};

(P, Q ∈ S[0. 5)).

This type of subsets are contained in S[0, 5).

Example 4.2: Let S([0, 8)) = {All subsets of the form $a_i \in [0, 8)$, [c, d), (e, f], (g, h] where c < d (or c > d) e < f (or e ≥ f), g < h (or g > h); a_i's are singletons} be the SSSS-interval set.

Example 4.3: Let S([0, 15)) = {Collection of all subsets of [0, 15) which includes the natural class of intervals} which is defined as SSSS-interval set.

We can build algebraic structures on them which is elaborately discussed in chapter II of this book.

Example 4.4: Let S([0, 21)) = {Collection of all subsets of [0, 21) which include intervals} be the SSSS-interval semigroup under min operation.

T_o, $T_{\cup}^{\min}$, $T_{\cap}^{\min}$ are the SSSS-interval semigroup topological spaces. $T_o = \{S' = S([0, 15)) \cup \{\phi\}, \cup, \cap\}$; $T_{\cup}^{\min} = \{S, \cup, \min\}$ and $T_{\cap}^{\min} = \{S', \cap, \min\}$.

Example 4.5: Let
S([0, 11)) = {Collection of all subsets and intervals} be the SSSS-interval semigroup. We have the three SSSS-interval topological spaces, T_o, $T_{\cup}^{\min}$ and $T_{\cap}^{\min}$.

All the three spaces are distinct. For take
A = {[0, 3), [7,9], 0.2, [0.001, 0], 1.8, 2.5, [5, 0.2]} and
B = {5, 2, [7, 2], [10, 1], [0, 0.001], [1, 0.2]} $\in T_o$.

A ∪ B = {[0, 3], [7,9], 0.2, [0.001, 0], 1.8, 2.5, [5,0.2, 5, 2, [7, 2], [10, 1], [0, 0.001], [1, 0.2]} = C.

$A \cap B = \phi = D$.

Clearly C, D $\in T_o$.

Let A, B $\in T_{\cup}^{\min}$.

$A \cup B = C$ and min {A, B} = [0, 3], 5, 0.2, [0.001, 0], [5, 0.2], 1.8, 2.5, [0, 2], 2, [2, 0.2], [7, 2], [1.8, 2], [2.5, 2], [0, 1], [7, 1], [1.8, 1], [2.5, 1], [0, 0.001], 0, [1, 0.2]} = E.

$E, C \in T_{\cup}^{\min}$.

Clearly T_o is different from $T_{\cup}^{\min}$.

Let $A, B \in T_{\cap}^{\min}$.

min {A, B} = E, $A \cap B = \{\phi\} = D$;

$E, D \in T_{\cap}^{\min}$

Thus $T_{\cap}^{\min}$ is different from T_o and $T_{\cup}^{\min}$.

Example 4.6: Let P = {S(C[0, 14)) = {Collection of all subsets which include the natural class of intervals}. Let T_o, $T_{\cup}^{\min}$ and $T_{\cap}^{\min}$ be the three SSSS-interval semigroup topological spaces associated with the semigroup P.

All these topological spaces have infinite cardinality and has infinite basis.

Example 4.7: Let M = {S(⟨[0, 3) ∪ I⟩) = {Collection of all subsets which include intervals, min} be the super subset interval semigroup. T_o, $T_{\cup}^{\min}$ and $T_{\cap}^{\min}$ be the SSSS-interval semigroup topological spaces associated with M.

Clearly if min operation is replaced in the semigroup by max we get the three semigroup topological spaces T_o, $T_{\cup}^{\max}$ and $T_{\cap}^{\max}$.

We will first illustrate this situation by an example or two.

Example 4.8: Let B = {S[0, 17), max} be the super special subset interval semigroup. T_o, $T_{\cup}^{max}$ and $T_{\cap}^{max}$ are the three SSSS-interval semigroup topological spaces associated with B. All the three spaces are distinct.

Example 4.9: Let B = {S[0, 4), max} be the super special subset interval semigroup. T_o, $T_{\cup}^{max}$ and $T_{\cap}^{max}$ be the three SSSS-interval semigroup topological spaces associated with the semigroup L.

Let A = {3, 24, 2.003, 1.7, [1, 2], [0.5, 3], [2, 0.001], [2, 3.7], [3.8, 0.5], 0.0007} and B = {0.3, 0.8, 3.2, 1.4, [0.2, 3], [1, 2], [3, 0.7], 1.7, [0.234, 1.27]} $\in T_o$.

A $\cup$ B = {3, 2.4, 2.003, 1.7, [0.5, 3], [2, 0.001], [2, 3.7], [3.8, 0.5], 0.007, 0.3, 0.8, 3.2, 1.4, [0.2, 3], [1, 2], [3, 0.7] [0.234, 1.2] ----- C

and A $\cap$ B = {[1, 2], 1.7} ---- D

Clearly C, D $\in T_o$.

Now A, B $\in T_{\cup}^{max}$;

A $\cup$ B = C and

max {A, B} = {3, 3.2, [2.4, 3], 2.4, [3, 2.4], 2.003, [2.003, 3], [3, 2.003], 1.7, [1.7, 3], [1.7, 2], [3, 1.7], [1.4, 2], [1,3] [1, 2], [3, 2], [3.8, 1.7], [0.8, 3], [3.8, 3.2], [3.8, 1.2], [1.4, 3], [0.5, 3], [1.7, 3], [3.8, 1.4], [2, 0.3], [2,0.8], [2, 1.4], [3.8, 3], [3.8,2], [3.8,0.7], [2, 3], 2, [2,1.7], [2, 1.2], [3.2, 3.7], [3,3.7], [3.8,0.5], [3.8,0.8]} ------ E

C, E $\in T_{\cup}^{max}$.

Let A, B $\in T_{\cap}^{max}$;

max {A, B] = E and $A \cap B$ = D and E, D $\in T_{\cap}^{\max}$.

It is clear all the three spaces are distinct.

Now we just replace max (or min) in the semigroup by $\times$ and give a few examples.

Example 4.10: Let {S([0, 9)), $\times$} = B be the super special interval semigroup. Let T_o, $T_{\cup}^{\times}$ and $T_{\cap}^{\times}$ be the three SSSS-interval semigroup topological spaces associated with the semigroup B.

Let
A = {(3,2), (5, 4), (1, 7), 0.33, 5.2, 3.1, (0.11, 2) (1.2, 0.1)}
and
B = {(1, 2), (3, 4), (7,0), 6.3, 1, (3.1, 2.1), 0.7, 0.8} $\in T_o$.

$A \cap B = \phi$ = C.

$A \cup B$ = {(3, 2), (5, 4), (1, 7), 0.33, 5.2, 3.1, (0.11, 2), (1.2, 0.1), (1, 2), (3, 4), (7, 0), 6.3, 1, (3.1, 2.1), 0.7, 0.8} = D.

Clearly D, C, $\in T_o$.

Let A, B $\in T_{\cup}^{\times}$; $A \cup B$ = D and

$A \times B$ = {(3, 4), (5, 8), (1, 5), (0.33, 0.66), (5.2,1.4), (3.1, 6.2), (0.11, 4), (1.2, 0.2), (9,8), (6,7), (3, 1), (0.99, 1.32), (6.6, 2.8), (0.3, 3.4), (0.33, 8), (3.6, 0.4), (3, 0), (8, 0), (7, 0), (2.31, 0), (1.4, 0), (3.7,0), (0.77,0), (8.4, 0) , (0.9, 3.6), (3.5, 7.2), (6.3, 8.1), 2.079, 2.76, 1.53, (0.693, 3.6), (7.56, 0.63), (3, 2), (5, 4), (1, 7), 0.33, 5.2, 3.1, (0.11, 2), (1.2, 0.1), (1.023,0.693), (0.3,4.2), (6.5,8.4), (3.1,5.7), (7, 12,1.92), (0.61, 6.51), (0.341, 9.2), (3.72, 0.021), (2.1, 1.4), (3.5, 2.8),(0.7,4.9), 0.231, 3.64, 2.17, (0.077, 1.4), (0.84, 0.07), (2.4, 1.6), (4, 3.2), (0.8, 5.6), 0.264, 4.16, 2.48, (0.088,1.6), (0.96, 0.08)} = E;

$D, E \in T_{\cup}^{\times}$.

Let $A, B \in T_{\cap}^{\times}$, $A \cap B = C$ and $A \times B = E$ and $C, E \in T_{\cap}^{\times}$.

Clearly all the three spaces are distinct.

Now we give some more examples of matrices using S([0, n)).

Example 4.11: Let
$T = \{(a_1, a_2, a_3) \mid a_i \in S([0, 10)), 1 \leq i \leq 3, \min\}$ be the super special subset interval semigroup. T_o, $T_{\cup}^{\min}$, $T_{\cap}^{\min}$ be SSSS-interval semigroup topological spaces associated with T.

Let A = ({(0, 7, (2, 5), 1, (7,0)}, {(0.3, 2.4), 3, 4.2, (1,0)}, {(0.5, 0.1), 2}) and

B = ({(0.4, (7, 2)}, {(0.3, 0), (1, 2)}, {0.5, 3}) $\in T_o$.

$A \cup B$ = {(0.7, (2, 5), 1, (7,0), 0.4, (7, 2)}, {(0.3, 2.4), 3, 4.2, (1, 0), (0.3, 0), (1, 2)}, {(0.5, 0.1), 2, 0.5, 3}) = C

$A \cap B = (\{\phi\},\{\phi\},\{\phi\}) = D$;

$C, D \in T_o$.

Let $A, B \in T_{\cup}^{\min}$

$A \cup B = C$ and

min {A, B} = {(0.4, 0.7), 1, (0.4, 0), (7,0)}, {(0.3, 0), (0.3, 2), (1, 2), (1, 0)}, {(0.5, 0.1), 0.5, 2}) = E;

$C, E \in T_{\cap}^{\min}$.

Let $A, B \in T_{\cup}^{\min}$;

min {A, B} = E and

$A \cap B = (\{\phi\}, \{\phi\}, \{\phi\}) = D$ and

$E, D \in T_{\cap}^{\min}$.

Thus all the three spaces are distinct.

Example 4.12: Let

$$L = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \end{bmatrix} \middle| a_i \in S([0, 10)), 1 \leq i \leq 7, \min \right\}$$

be the super special subset interval semigroup.

T_o, $T_{\cup}^{\min}$ and $T_{\cap}^{\min}$ are the SSSS-interval semigroup topological spaces and all are distinct.

Example 4.13: Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \middle| a_i \in S([0, 4)), 1 \leq i \leq 4, \max \right\}$$

be the super subset interval semigroup.

Let T_o, $T_{\cup}^{\max}$ and $T_{\cap}^{\max}$ be the three SSSS-interval topological spaces associated with M.

We show all the three spaces are distinct.

$$\text{Let } A = \begin{bmatrix} \{0.3,(2,0.4)\} & \{1,0,(1,2)\} \\ \{2.3,(0.1,2)(0,3)\} & \{(0,2),0,(0.7,0)\} \end{bmatrix}$$

and

$$B = \begin{bmatrix} \{(2,0),(1,2),3.2\} & \{1,2,(0,1)\} \\ \{0,1.2,1.3,(2,3)\} & \{0,1.3,(0.3,2)\} \end{bmatrix} \in T_o$$

$$A \cap B = \begin{bmatrix} \{\phi\} & \{1\} \\ \{\phi\} & \{0\} \end{bmatrix} = D.$$

$A \cup B =$

$$\begin{bmatrix} \{0.3,(2,0.4),(2,0),(1,2),3.2\} & \{1,0,(1,2),2,(0,1)\} \\ \{2.3,(0.1,2),(0,3),0,1.2,1.3,(2,3)\} & \{0,1.3,(0,2),(0.3,2),(0.7,0)\} \end{bmatrix}$$

$= D$; $C, D \in T_o$.

Let $A, B \in T_{\cup}^{\max}$

$\max \{A, B\} =$

$$\begin{bmatrix} \{(2,0.3),(1,2),3.2,(2,0.4),2\} & \{1,2,(0,1),(1,2)\} \\ \{2.3,(2.3,3),(0.1,2),(0,3),(1.2,2), & \{(0,2),(0.7,0),1.3,(0.3,2), \\ (1.3,2),(1.2,3),(2,3),(1.3,3)\} & (1.3,2),(0.7,2)\} \end{bmatrix}$$

$= E$;

$E, D \in T_{\cup}^{\max}$.

Let $A, B \in T_{\cap}^{\max}$ and $E, C \in T_{\cap}^{\max}$.

Clearly all the three spaces are distinct and are of infinite cardinality with infinite basis.

Example 4.14: Let

$$B = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \end{bmatrix} \middle| a_i \in S([0, 12)), 1 \leq i \leq 10, \max \right\}$$

be the super subset special interval semigroup.

Let T_o, $T_{\cup}^{\max}$, $T_{\cap}^{\max}$ be the SSSS-interval topological spaces associated with B, all the three spaces are distinct.

Example 4.15: Let
$S = \{(a_1, a_2, a_3, a_4) \mid a_i \in S([0, 24)), 1 \leq i \leq 4, \times\}$ be the super subset special interval semigroup.

Let T_o, $T_{\cup}^{\times}$ and $T_{\cap}^{\times}$ are the SSSS-interval semigroup topological spaces.

$T_{\cup}^{\times}$ and $T_{\cap}^{\times}$ has subspaces such that $A \times B = \{(0, 0, 0, 0)\}$.

Let $P_{\cup}^{\times} = \{(a, b, 0, 0) \mid a, b \in S([0, 24)), \times, \cup\} \subseteq T_{\cup}^{\times}$

and $B_{\cap}^{\times} = \{(0, 0, 0, c) \mid c \in S([0, 24)), \times, \cap\} \subseteq T_{\cap}^{\times}$ are subspaces such that $B_{\cap}^{\times} \times P_{\cup}^{\times} = \{(0, 0, 0, 0)\}$.

Thus in case of $T_{\cap}^{\times}$ and $T_{\cup}^{\times}$, we have pairs of subspaces $A, B \in T_{\cap}^{\times}$ (or $T_{\cup}^{\times}$) such that $A \times B = \{(0)\}$.

Example 4.16: Let

$$S = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \end{bmatrix} \middle| \, a_i \in S([0, 12)), 1 \leq i \leq 7, \times_n \right\}$$

be the super subset special interval semigroup.

Let T_o, $T_{\cup}^{\times_n}$ and $T_{\cap}^{\times_n}$ be the SSSS-interval semigroup topological spaces. Clearly $T_{\cup}^{\times_n}$ and $T_{\cap}^{\times_n}$ have pairs of subspaces such that their product is zero.

Next we define the notion of SSSS-interval pseudo group topological spaces by some examples.

Example 4.17: Let G = {S([0, 8)), +} be the super subset special pseudo group. T_o, $T_{\cup}^{+}, T_{\cap}^{+}$ be the SSSS-interval pseudo group topological spaces associated with G.

Let A, B $\in T_o$ where

$A = \{(6, 2), (3.1, 2.5), (7, 1.6), 0.34, 0.25\}$ and
$B = \{6.31, 7.2, (3, 2.5), (4.5, 2.7), 0\} \in T_o$.

$A \cap B = \{\phi\} = D$.

$A \cup B = \{6.31, 7.2, (3, 2.5), (4.5, 2.7), 0, (6, 2), (3.1, 2.5), (7, 1.6), 0.34, 0.25\} = E$;

D, E $\in T_o$. For A, B $\in T_{\cup}^{+}$;

A + B={(4.31, 0.31), (5.2,1.2), (5.31,7.91,(1, 4.5),(2.5, 4.7), (6.4, 0.8), (6, 2), (1.41, 0.31), (2, 4.1), (2.3,1.7), (3.5,4.3), (6.1, 5), (7, 1.6), (7.6, 5.2), (3.1, 2.5), 6.65, 7.54, (3.34, 2.84), (4.84, 3.04), 0.34, 6.56, 7.45, (3.25, 2.75), (4.75,2.95), 0.25} = F;

$A \cup B = E$; $E, F \in T_{\cup}^{+}$.

Let $A, B \in T_{\cap}^{+}$ with

$A + B = F$ and $A \cap B = D$ further
$F, D \in T_{\cap}^{+}$.

The three SSSS-interval pseudo group topological spaces are distinct.

Example 4.18: Let
$S = \{(a_1, a_2, a_3) \mid a_i \in S([0, 12]), 1 \leq i \leq 3, +\}$ be the super subset interval pseudo group.

Let T_o, $T_{\cup}^{+}$ and $T_{\cap}^{+}$ be the three SSSS-interval pseudo group topological spaces. They are distinct.

Example 4.19: Let

$$S = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} \middle| a_i \in S([0, 3)), 1 \leq i \leq 4, +\right\}$$

be the super subset interval pseudo group.

Let T_o, $T_{\cup}^{+}$ and $T_{\cap}^{+}$ be the three SSSS-interval pseudo topological spaces.

To every x $\in$ $T_{\cup}^{+}$ (or $T_{\cap}^{+}$) there is a y such that $x + y = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$.

Example 4.20: Let S ={S([0,10)), min, max} be the super subset interval semiring. T_o, $T_{\cup}^{\min}$ $T_{\cup}^{\max}$, $T_{\cap}^{\min}$, $T_{\cap}^{\max}$ and T_s are defined as the SSSS-interval semiring topological spaces and all the six spaces are distinct.

Let A = {0.5, 5, 3.4, 6.3, 7, 2} and
B = {5, 0.5, 2.4, 2, 9, 2.5, 8} $\in$ T_o.

A $\cup$ B = {0.5, 5, 3.4, 6.3, 7, 2, 2.4, 2, 9, 2.5, 8} = C,

A $\cap$ B = {0.5, 5, 2} = D.

Both A $\cup$ B and A $\cap$ B $\in$ T_o.

Let A, B $\in$ $T_{\cup}^{\min}$; A $\cup$ B = C and
min {A, B} = {0.5, 2.4, 2, 2.5, 3.4, 5, 6.3, 7} = E.

Clearly C, E $\in$ $T_{\cup}^{\min}$.

We see T_o is different from $T_{\cup}^{\min}$.

Let A, B $\in$ $T_{\cap}^{\min}$,

min {A, B} = E and

A $\cap$ B = D; E, D $\in$ $T_{\cap}^{\min}$.

$T_{\cap}^{\min}$ is different from T_o and $T_{\cup}^{\min}$.

Let A, B $\in$ $T_{\cup}^{\max}$;

max {A, B} {5, 0.5, 2.4, 2, 9, 2.5, 8, 3.4, 6.3, 7} = F and

$A \cup B = C$; C and F $\in T_{\cup}^{\max}$.

$T_{\cup}^{\max}$ is different from T_o, $T_{\cup}^{\min}$ and $T_{\cap}^{\min}$.

Consider A, B $\in T_{\cap}^{\max}$;

max {A, B} = F and

$A \cap B = D$; F, D $\in T_{\cap}^{\max}$.

$T_{\cup}^{\min}$, $T_{\cap}^{\min}$ and $T_{\cap}^{\max}$ are distinct.

Consider A, B $\in T_s$.

min {A, B} = E and max {A, B} = F.

E, F $\in T_s$ and T_s is different from T_o, $T_{\cup}^{\max}$, $T_{\cap}^{\max}$, $T_{\cup}^{\min}$, and $T_{\cap}^{\min}$.

Further all the spaces have infinite cardinality and infinite basis.

Example 4.21: Let

$$M = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \middle| a_i \in S([0, 6)), 1 \leq i \leq 3, \min, \max \right\}$$

be the super subset interval semiring. Let T_o, $T_{\cup}^{\min}$, $T_{\cap}^{\min}$, $T_{\cup}^{\max}$, $T_{\cap}^{\max}$ and T_s be the six SSSS-interval semiring topological spaces.

Clearly all the six spaces are distinct and are infinite.

Example 4.22: Let

$$S = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \\ a_{17} & a_{18} & a_{19} & a_{20} \end{bmatrix} \right| a_i \in S([0, 15)),$$

$$1 \leq i \leq 20, \text{min, max}\}$$

be the super subset interval semiring. Let T_o, T_s, $T_{\cup}^{\min}$, $T_{\cap}^{\min}$, $T_{\cup}^{\max}$ and $T_{\cap}^{\max}$ are the six distinct SSSS-interval semiring topological spaces associated with S. Infact these have infinite basis.

Example 4.23: Let S = {S([0, 12)), $\times$, min} be the super subset interval pseudo semiring. T_o, $T_{\cup}^{\min}$, $T_{\cap}^{\min}$, $T_{\cup}^{\times}$, $T_{\cap}^{\times}$ and T_s are the six SSSS-interval pseudo semiring topological spaces.

All of them are distinct.

Let A = {0.3, 4.1, 3.2, 2, 7, 5.4, (1, 2), (0, 0.7), (3.2, 1)} and B = {7, 5, 3, 2, (1, 4)} $\in T_o$.

$A \cup B$ = {0.3, 4.1, 3.2, 2, 7, 5.4, (1, 2), (0, 0.7), (3.2, 1), 5, 3, (1, 4)} = C and

$A \cap B$ = {2, 7} = D; C, D $\in T_o$.

Let A, B $\in T_{\cup}^{\min}$

$A \cup B$ = C and min {A, B} = {0.3, 3, 2, (1, 4), (1, 3.2), (1, 2), 7, 5.4, (1, 2), (0, 0.7)} = E.

$C, E \in T_{\cup}^{\min}$.

Let $A, B \in T_{\cap}^{\min}$;

$A \cap B = D$ and $\min \{A, B\} = E$ and

$D, E \in T_{\cap}^{\min}$.

Consider $A, B \in T_{\cup}^{\times}$; $A \cup B = C$ and

$A \times B = \{2.1, 1.5, 0.9, 0.6, 0.3, 8.2, (0.3, 1.2), 4.7, 8.5, (4.1, 4.4), 10.4, 4, 9.6, 6.4, (3.2, 0.8), 2, 10, 6, (2, 8), 1, 11, 9, (7, 4), 1.8, 3, 4.2, 10.8, (5.4, 9.6), (7, 2), (510), (36), (2, 4), (1, 8), (0, 4.9), (0, 3.5), (0, 3.1), (0, 1.4), (0, 2.8)\} = F$.

$C, F \in T_{\cup}^{\times}$.

Consider $A, B \in T_{\cap}^{\times}$;

$A \cap B = D$ and $A \times B = F$

$D, F \in T_{\cap}^{\times}$.

Finally for $A, B \in T_s$.

$A \times B = F$ and $\min \{A, B\} = E$.

$E, F \in T_s$.

It is easily verified all the 6 SSSS-interval pseudo semiring topological spaces are distinct and has infinite basis.

Example 4.24: Let
$M = \{(a_1, a_2, a_3, a_4, a_5) \mid a_i \in S([0, 17)), 1 \leq i \leq 5, \min, \times\}$ be the super subset interval pseudo semirings.

Clearly T_o, $T_{\cup}^{\min}$, $T_{\cap}^{\min}$, $T_{\cup}^{\times}$, $T_{\cap}^{\times}$ and T_s are 6 different SSSS-interval pseudo semiring topological spaces associated with the pseudo semiring M.

Example 4.25: Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \end{bmatrix} \middle| \; a_i \in S([0, 11)), \right.$$

$$\left. 1 \le i \le 15, \min, \times_n \right\}$$

be the super subset interval pseudo semiring T_o, $T_{\cup}^{\times_n}$, $T_{\cap}^{\times_n}$, $T_{\cup}^{\min}$, $T_{\cap}^{\min}$ and T_s are the six distinct SSSS-interval pseudo semiring topological spaces associated with the super subset interval pseudo semiring M.

These pseudo semiring SSSS-interval topological spaces enjoy a special property namely; T_s, $T_{\cup}^{\times}$ and $T_{\cap}^{\times}$ have pairs of subspaces S_1 and S_2 such that $S_1 \times S_2 = \{(0)\}$.

Next we proceed onto discuss about pseudo ring SSSS-topological spaces associated with a super special subset interval pseudo ring by examples.

Example 4.26: Let R = {S ([0, 24)), +, ×} be the super special subset interval pseudo ring topological spaces associated with the pseudo ring R.

Let A = {(0, 3), (4, 0.1), 10, 0.7, 6.5} and
B = {10, (0, 3), 8, 0.4, 2, (0.5, 0)} $\in T_o$.

$A \cup B$ = {10, (0, 3), (4, 0.1), 0.7, 6.5, 8, 0.4, 2, (0.5, 0)} = C and $A \cap B$ = {(0, 3), 10} = D and

D, C $\in T_o$.

Let A, B $\in T_{\cup}^{+}$;

A + B = {(10, 13), (14, 10.1), 20, 10.7, 16.5, (0, 6), (4, 3.1), (6.5, 9.5), (0.7, 3.7), (8, 11), (12, 8.1), 18, 8.7, 14.5, (0.4, 3.4), (4.4, 0.5), 10.4, 1.1, (4.5, 0.1), 6.9, (2, 5), (6, 2.1), (0.5,3), 12, 2.7, 8.5, (10.5, 10), (7, 6.5), (1.2, 0.7)} – E and A $\cup$ B = C, E and C $\in T_{\cup}^{+}$.

T_o and $T_{\cup}^{+}$ are different.

Let A, B $\in T_{\cap}^{+}$;

A $\cap$ B = D and A + B = E further

D, E $\in T_{\cap}^{+}$.

Clearly $T_{\cap}^{+}$ is different from T_o and $T_{\cup}^{+}$.

A $\cap$ B = D and

A $\times$ B = {(0,6), (16, 1), 4, 0, 7, 17, (0, 0.3), (0, 2.1), (0, 19.5), (8, 0.8), 8, 5.6, (0, 1.2), (1.6, 0.04), 0.28, 2.6, (0, 6), (8, 0.2), 20, 1.4, 13, (2, 0),(5, 0), (0.35, 0), (3.25, 0)} = F.

D, E $\in T_{\cap}^{+}$.

$T_{\cap}^{+}$ is different from T_o, $T_{\cup}^{+}$ and $T_{\cap}^{+}$.

Consider A, B $\in T_{\cup}^{\times}$ and

A $\cup$ B = C and A $\times$ B = F,

C, F $\in T_{\cup}^{\times}$.

$T_{\cup}^{\times}$ is different from T_o, $T_{\cup}^{+}$, $T_{\cap}^{+}$ and $T_{\cap}^{\times}$.

Example 4.27: Let
$S = \{(a_1, a_2, a_3) \mid a_i \in S([0, 15)), 1 \leq i \leq 3, +, \times\}$ be the SSSS-interval row matrix pseudo ring. T_o, $T_{\cup}^{+}$, $T_{\cap}^{+}$, $T_{\cap}^{\times}, T_{\cup}^{\times}$ and T_s be the six super subset interval semiring topological spaces associated with the semiring $M = \{S, \cup, \cap\}$.

$T_{\cap}^{\times}, T_{\cup}^{\times}$ and T_s has pairs of subspaces which are such that their product is the zero space.

Example 4.28: Let $S = \left\{\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{10} \end{bmatrix} \middle| a_i \in S([0, 16)), 1 \leq i \leq 10, +, \times_n\right\}$

be the super subset interval pseudo semiring.

Let $M = \{S_1, \cup, \cap\}$ be the ring subring T_o, $T_{\cup}^{+}, T_{\cap}^{+}, T_{\cup}^{\times_n}, T_{\cap}^{\times_n}$ and T_s be the pseudo SSSS-interval ring semiring topological space associated with the semiring M.

Example 4.29: Let

$$S = \left\{\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \end{bmatrix} \middle| a_i \in S([0, 10)), 1 \leq i \leq 15, +, \times\right\}$$

be the super subset interval pseudo semiring.

Let $M = \{S, \cup, \cap\}$ be the ring subring.

Let T_o, $T_{\cup}^{+}, T_{\cap}^{+}, T_{\cap}^{\times}, T_{\cup}^{\times}$ and T_s be the SSSS-interval ring semiring pseudo topological spaces associated with the ring semiring M.

All properties associated with these topological spaces can be derived by making appropriate modifications.

Now we give a few problems for the reader.

Problems:

1. Obtain some special features enjoyed by these SSSS-semigroup topological spaces.

2. Compare a usual topological space with the SSSS-semigroup topological space.

3. Prove there does not exist a SSSS-semigroup topological space with a finite basis.

4. Let S = {S([0, 24)) = {Collection of all subsets of [0, 24) including the natural class of intervals}, min} be the super subset semiring. Let T_o, $T_{\cup}^{\min}$, $T_{\cap}^{\min}$ be the SSSS-interval semigroup topological space associated with S.

   (i) Prove T_o, $T_{\cup}^{\min}$, $T_{\cap}^{\min}$ are distinct spaces.
   (ii) Show they have infinite basis.
   (iii) Find a basis of T_o, $T_{\cup}^{\min}$ and $T_{\cap}^{\min}$.
   (iv) Can a basis of T_o serve as a basis of $T_{\cup}^{\min}$ and $T_{\cap}^{\min}$?
   (v) Prove T_o, $T_{\cup}^{\min}$ and $T_{\cap}^{\min}$ has subspaces of finite cardinality.
   (vi) Obtain any other special feature enjoyed by T_o, $T_{\cup}^{\min}$ and $T_{\cap}^{\min}$.

5. Let S = {S([0, 24)), max} be the super subset interval semigroup T_o, $T_{\cup}^{max}$ and $T_{\cap}^{max}$ be the SSSS-interval semigroup topological spaces associated with this S.

   Study questions (i) to (vi) of problem 4 for this S.

6. Let P = $\{(a_1, a_2, \ldots, a_6) \mid a_i \in S([0, 12)), 1 \leq i \leq 6; \min\}$ be the super subset interval semigroup T_o, $T_{\cup}^{min}$ and $T_{\cap}^{min}$ be the SSSS-interval semigroup topological space.

   Study questions (i) to (vi) of problem 4 for this P.

7. Let M = $\left\{\left.\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{12} \end{bmatrix}\right| a_i \in S([0, 13)), 1 \leq i \leq 12, \max\right\}$ be the super interval subset semigroup. T_o, $T_{\cup}^{max}$ and $T_{\cap}^{max}$ be the SSSS-interval semigroup topological space.

   Study questions (i) to (vi) of problem 4 for this M.

8. Let W = $\left\{\left.\begin{pmatrix} a_1 & a_2 & \cdots & a_8 \\ a_9 & a_{10} & \cdots & a_{16} \\ a_{17} & a_{18} & \cdots & a_{24} \\ a_{25} & a_{26} & \cdots & a_{32} \end{pmatrix}\right| a_i \in S([0, 18)), 1 \leq i \leq 32, \min\right\}$ be the super subset interval semigroup. T_o, $T_{\cup}^{min}$ and $T_{\cap}^{min}$ be the SSSS-topological semigroup space.

   Study questions (i) to (vi) of problem 4 for this W.

9. Let $T = \left\{ \begin{pmatrix} a_1 & a_2 & \ldots & a_9 \\ a_{10} & a_{11} & \ldots & a_{18} \end{pmatrix} \middle| a_i \in S([0, 21)), \right.$

$1 \leq i \leq 18, \times\}$ be the super subset interval semigroup. T_o, $T_{\cup}^{\times}$ and $T_{\cap}^{\times}$ be the SSSS-interval semigroup topological space.

(i) Study questions (i) to (vi) of problem 4 for this T.

(ii) Show the spaces $T_{\cup}^{\times}$ and $T_{\cap}^{\times}$ have pairs of subspaces A, B with $A \times B = ((0))$.

10. Let M = {S([0, 18)), min, max} be the super subset interval semiring. Let T_o, $T_{\cup}^{\min}$, $T_{\cap}^{\max}$, $T_{\cap}^{\min}$, $T_{\cup}^{\max}$ and T_s be the six SSSS-interval semiring topological spaces related to M.

(i) Prove all the 6 topological spaces are distinct.

(ii) Can a basis of T_o (or $T_{\cup}^{\max}$, $T_{\cup}^{\min}$ …) be a basis for the rest of the five spaces? Justify your claim.

(iii) Prove T_o, $T_{\cup}^{\max}$, …, T_s have subspaces of finite cardinality hence a finite basis.

(iv) Study or obtain any other special feature associated with these spaces.

11. Let $M = \{(a_1, a_2, \ldots, a_7) \mid a_i \in S([0, 15)); 1 \leq i \leq 7$, min, max} be the super subset interval semiring.

Let T_o, $T_{\cup}^{\min}$, $T_{\cap}^{\min}$, $T_{\cup}^{\max}$, $T_{\cap}^{\max}$ and T_s be the six SSSS-interval semiring topological spaces associated with M.

Study questions (i) to (iv) of problem 10 for this M.

12. Let N = $\left\{\left.\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_9 \end{bmatrix}\right| a_i \in S([0, 11)), 1 \leq i \leq 9, \text{min, max}\right\}$ be the super interval semigroup. T_o, $T_\cup^{\min}$, $T_\cap^{\min}$, $T_\cup^{\max}$, $T_\cap^{\max}$ and T_s be the SSSS-interval semigroup topological spaces associated with semiring N.

    Study questions (i) to (iv) of problem 10 for this N.

13. Let S = $\left\{\left.\begin{pmatrix} a_1 & a_2 & \cdots & a_{14} \\ a_{15} & a_{16} & \cdots & a_{28} \\ a_{29} & a_{30} & \cdots & a_{42} \\ a_{43} & a_{44} & \cdots & a_{56} \end{pmatrix}\right| a_i \in S([0, 26)),\right.$ $1 \leq i \leq 56$, min, max$\}$ be the super subset interval semigroup. T_o, $T_\cup^{\min}$, $T_\cap^{\min}$, $T_\cup^{\max}$, $T_\cap^{\max}$ and T_s be the SSSS-interval semigroup topological spaces related to the semiring S.

    Study questions (i) to (iv) of problem 10 for this S.

14. Let S = {S([0, 19)), ×, min} be the super subset pseudo semiring. T_o, $T_\cup^{\times}$, $T_\cap^{\times}$, $T_\cup^{\min}$, $T_\cap^{\min}$, $T_\cup^{\max}$, $T_\cap^{\max}$ and T_s be the SSSS-interval semiring pseudo topological semiring pseudo topological spaces associated with the semiring S.

    Study questions (i) to (iv) of problem (10) for this spaces associated with the pseudo semiring S.

15. Let $S_1 = \{(a_1, a_2, \ldots, a_8) \mid a_i \in S([0, 12));\ 1 \leq i \leq 8, \min, \times\}$ be the super subset interval pseudo semiring T_o, $T_{\cup}^{\min}$, $T_{\cap}^{\min}$, $T_{\cup}^{\times}, T_{\cap}^{\times}$ and T_s be the SSSS-interval pseudo semiring topological spaces associated with the pseudo semiring S_1.

    (i) Study questions (i) to (iv) of problem 10 for these pseudo spaces associated with this matrix semiring S_1.

    (ii) Prove the pseudo semiring topological spaces $T_{\cup}^{\times}, T_{\cap}^{\times}$ and T_s has pairs of topological subspaces such that their product is a zero space.

16. Let $T = \left\{ \begin{pmatrix} a_1 & a_{11} & a_{21} & a_{31} \\ a_2 & a_{12} & a_{22} & a_{32} \\ \vdots & \vdots & \cdots & \vdots \\ a_{10} & a_{20} & a_{30} & a_{40} \end{pmatrix} \middle| \ a_i \in S([0, 15)), \right.$

    $1 \leq i \leq 40, \times_n, \min\}$ be the super subset interval pseudo semiring.

    Let T_o, $T_{\cup}^{\times_n}, T_{\cap}^{\times_n}, T_{\cup}^{\min}$, $T_{\cap}^{\min}$ and T_s be the SSSS-interval pseudo semiring topological spaces.

    Study questions (i) and (ii) of problem 15 for these pseudo spaces related to the pseudo semiring T.

17. Compare the SSSS-interval semiring topological spaces with SSSS-interval pseudo semiring topological spaces.

18. Let $S = \{S([0, 24)), \times, +\}$ be the super subset interval pseudo ring T_o, $T_{\cup}^{+}, T_{\cap}^{\times}$ $T_{\cap}^{+}, T_{\cup}^{\times}$ and T_s be the six SSSS-interval pseudo ring topological spaces associated with the super interval subset pseudo ring S.

(i) Prove all the spaces are of infinite cardinality having an infinite basis.

(ii) Prove these spaces have subspaces of finite cardinality and hence finite basis.

(iii) Prove $T_{\cap}^{\times}, T_{\cup}^{\times}$ and T_s has pairs of subspaces A, B such that $A \times B = \{(0)\}$.

(iv) Can a common subset serve as a basis for all the 6 spaces?

19. Let $S = \{(a_1, a_2, \ldots, a_8) \mid a_i \in S([0,24)), 1 \leq i \leq 8, +, \times\}$ be the super subset interval pseudo ring. T_o, $T_{\cup}^{+}, T_{\cap}^{+}$, $T_{\cap}^{\times}, T_{\cup}^{\times}$ and T_s be the SSSS-interval pseudo topological spaces related with the pseudo ring S.

Study questions (i) to (iv) of problem (18) for this pseudo ring S.

20. Let $R_1 = \left\{\left[\begin{array}{c} a_1 \\ a_2 \\ \vdots \\ a_9 \end{array}\right] \middle| a_i \in S([0, 11)), 1 \leq i \leq 9, +, \times_n\right\}$

be the super subset interval pseudo ring. T_o, $T_{\cup}^{+}, T_{\cap}^{+}$, $T_{\cap}^{\times}, T_{\cup}^{\times}$ and T_s. be the SSSS-pseudo ring semiring topological space, related to the pseudo ring R_1.

Study questions (i) to (iv) of problem (18) for this pseudo ring R_1.

21. Let M = $\left\{\left(\begin{array}{c|ccc|c|cc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ a_8 & \ldots & \ldots & \ldots & \ldots & \ldots & a_{14} \end{array}\right) \middle| \, a_i \in \right.$ S([0, 14)), $1 \leq i \leq 14$, +, $\times_n\}$ be the super subset interval pseudo ring. T_o, $T_{\cup}^{+}, T_{\cap}^{+}$, $T_{\cap}^{\times}, T_{\cup}^{\times}$ and T_s be the SSSS-pseudo ring semiring topological space, related to the pseudo ring M.

    Study questions (i) to (iv) of problem (18) for this pseudo ring M.

22. Can there be a pseudo ring R so that al the six SSSS-pseudo topological spaces have the same basis?

23. Does there exist a pseudo semiring for which all the 6 associated SSSS-interval topological spaces have the same basis?

24. Does there exist a semiring S for which all the 6 associated SSSS-interval topological spaces have the same basis?

25. Does there exist a pseudo ring for which none of the six SSSS-pseudo topological spaces have a common basis?

26. Does there exist a pseudo semiring R for which all the SSSS-interval topological pseudo spaces related to R have distinct basis?

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On India's 60th Independence Day, Dr.Vasantha was conferred the Kalpana Chawla Award for Courage and Daring Enterprise by the State Government of Tamil Nadu in recognition of her sustained fight for social justice in the Indian Institute of Technology (IIT) Madras and for her contribution to mathematics. The award, instituted in the memory of Indian-American astronaut Kalpana Chawla who died aboard Space Shuttle Columbia, carried a cash prize of five lakh rupees (the highest prize-money for any Indian award) and a gold medal.

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In this book for the first time authors introduce a special type of topological spaces using interval [o, n). Several of the properties enjoyed by these special spaces are analysed. Over hundred problems are suggested some of which are open conjectures. This book gives a new perspective to topological spaces

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